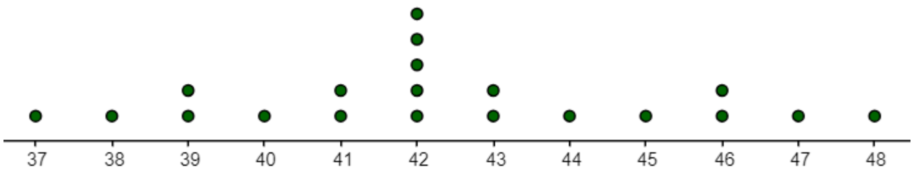
Mostly Harmless Statistics

Student Solutions Manual

Chapter 1 Exercises

1. The dotplot shows the height of some 5-year-old children measured in inches. Use the distribution of heights to find the approximate answer to the question, “How many inches tall are 5-year-olds?”



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> “‘Alright,’ he said, ‘but where do we start? How should I know?
> They say the Ultimate Answer or whatever is Forty-two, how am I supposed to know what the question is? It could be anything. I mean, what's six times seven?’
> Zaphod looked at him hard for a moment. Then his eyes blazed with excitement.
> ‘Forty-two!’ he cried.”
> (Adams, 2002)

3. What are statistics? Answer c)
   a) A question with a variety of answers.
   b) A way to measure the entire population.
   c) The science of collecting, organizing, analyzing and interpreting data.
   d) A question from a survey.

5. Which of the following are statistical questions? Select all that apply. Answer b, c, e
   a) How old are you?
   b) What is the weight of a mouse?
   c) How tall are all 3-year-olds?
   d) How tall are you?
   e) What is the average blood pressure of adult men?

   The questions "How old are you?" and "How tall are you?" are not statistical questions because the answers to these questions will not vary. There is ONE correct answer, as opposed to many answers that can then be used to find an average, proportion, range, etc.

Questions 7-9:

Some helpful definitions:
Individual is a person or object that you are interested in finding out information about.
Variable is the measurement or observation of the individual.

Population is the total set of all the observations that are the subject of a study.
Sample a subset from the population.
Parameter a number calculated from the population.
Statistic a number calculated from the sample.

7. Suppose you want to estimate the percentage of videos on YouTube that are cat videos. It is impossible for you to watch all videos on YouTube so you use a random video picker to select 1,000 videos for you. You find that 2% of these videos are cat videos. Determine which of the following is an observation, a variable, a sample statistic, or a population parameter.
   a) Percentage of all videos on YouTube that are cat videos. Population Parameter
   b) A video in your sample. Observation
   c) 2% Sample Statistic
   d) Whether a video is a cat video. Variable

9. The 2010 General Social Survey asked the question, "After an average workday, about how many hours do you have to relax or pursue activities that you enjoy?" to a random sample of 1,155 Americans. The average relaxing time was found to be 1.65 hours. Determine which of the following is an individual, a variable, a sample statistic, or a population parameter.
   a) Average number of hours all Americans spend relaxing after an average workday. Population Parameter
   b) 1.65 Sample Statistic
   c) An American in the sample. Individual
   d) Number of hours spent relaxing after an average workday. Variable

11. In a study, the sample is chosen by separating all cars by size, and selecting 10 of each size grouping. What is the sampling method? Stratified

13. In a study, the sample is chosen by asking people on the street. What is the sampling method? Convenience

15. In a study, the sample is chosen by surveying every 3rd driver coming through a tollbooth. What is the sampling method? Systematic

17. State whether each study is observational or experimental.
   a) You want to determine if cinnamon reduces a person's insulin sensitivity. You give patients who are insulin sensitive a certain amount of cinnamon and then measure their glucose levels. Experimental
   b) A researcher wants to evaluate whether countries with lower fertility rates have a higher life expectancy. They collect the fertility rates and the life expectancies of countries around the world. Observational
   c) A researcher wants to determine if diet and exercise together helps people lose weight over just exercising. The researcher solicits volunteers to be part of the study, and then randomly assigns the volunteers to be in the diet and exercise group or the exercise only group. Experimental
   d) You collect the weights of tagged fish in a tank. You then put an extra protein fish food in water for the fish and then measure their weight a month later. Experimental

An **observational study** is when the investigator collects data by observing, measuring, counting, watching, or asking questions. The investigator does not change anything.

An **experimental study** is when the investigator changes a variable or imposes a treatment to determine its effect.

19. Researchers studying the relationship between honesty, age and self-control conducted an experiment on 160 children between the ages of 5 and 15. Participants reported their age, sex, and whether they were an only child or not. The researchers asked each child to toss a fair coin in private and to record the outcome (white or black) on a paper sheet, and said they would only reward children who report white. Half the students were explicitly told not to cheat and the others were not given any explicit instructions. In the no instruction group, the probability of cheating was found to be uniform across groups based on child's characteristics. In the group that was explicitly told to not cheat, girls were less likely to cheat, and while rate of cheating did not vary by age for boys, it decreased with age for girls. [Alessandro Bucciol and Marco Piovesan. "Luck or cheating? A field experiment on honesty with children." In: Journal of Economic Psychology 32.1 (2011), pp. 73–78.] In this study, identify the variables. Select all that apply. Answers a, b, d, f, g
    a) Age
    b) Sex
    c) Paper Sheet
    d) Cheated or Not
    e) Reward for White Side of Coin
    f) White or Black Side of Coin
    g) Only Child or Not

A **variable** is the measurement or observation of an individual.

Nominal data is categorical data that has no order or rank, for example the color of your car, ethnicity, race, or gender.

Ordinal data is categorical data that has a natural order to it for example, year in school (freshman, sophomore, junior, senior), a letter grade (A, B, C, D, F), the size of a soft drink (small, medium, large) or **Likert scales**.

Interval data is numeric where there is a known difference between values, but zero does not mean "nothing." Interval data is ordinal, but you can now subtract one value from another and that subtraction makes sense. You can do arithmetic on this data. For example, Fahrenheit temperature, 0° is cold but it does not mean that no temperature exists. Time, dates and IQ scores are other examples.

Ratio data is numeric data that has a true zero, meaning when the variable is zero nothing is there. Most measurement data are ratio data. Some examples are height, weight, age, distance, or time running a race.

21. Select the measurement scale Nominal, Ordinal, Interval or Ratio for each scenario.

a) Temperature in degrees Kelvin. Ratio
b) Eye color. Nominal
c) Year in school (freshman, sophomore, junior, senior). Ordinal
d) The weight of a hummingbird. Ratio
e) The height of a building. Ratio
f) The amount of iron in a person's blood. Ratio
g) A person's gender. Nominal

A **qualitative variable** is a word or name that describes a characteristic (quality) of the individual.

A **quantitative or numerical variable** is a number (quantity), something that can be counted or measured from the individual.

23. State which type of variable each is, qualitative or quantitative?

a) The height of a giraffe. Quantitative
b) A person's race. Qualitative
c) Hair color. Qualitative
d) A person's ethnicity. Qualitative
e) Year in school (freshman, sophomore, junior, senior). Qualitative

Discrete data can only take on particular values like integers. Discrete data are usually things you count.

Continuous data can take on any value. Continuous data are usually things you measure.

25. State whether the variable is discrete or continuous.

a) Temperature in degrees Celsius. Continuous
b) The number of cars for sale at a car dealership. Discrete
c) The time it takes to run a marathon. Continuous
d) The amount of mercury in a tuna fish. Continuous
e) The weight of a hummingbird. Continuous

A **cross-sectional study** has been conducted when data is observed, measured, or collected at one point in time.

A **retrospective study** has been conducted when data is collected from the past using records, interviews, and other similar artifacts.

A **prospective study** has been conducted when data is collected in the future from groups sharing common factors.

27. Which type of sampling method is used for each scenario, Random, Systematic, Stratified, Cluster or Convenience?
    a) The quality control officer at a manufacturing plant needs to determine what percentage of items in a batch are defective. The officer chooses every 15^{th} batch off the line and counts the number of defective items in each chosen batch. Systematic
    b) The local grocery store lets you survey customers during lunch hour on their preference for a new bottle design for laundry detergent. Convenience
    c) Put all names in a hat and draw a certain number of names out. Random
    d) The researcher randomly selects 5 hospitals in the U.S. then measures the cholesterol level of all the heart attack patients in each of those hospitals. Cluster

A **Simple Random Sample** is a sample collected from the population so that every sample of the same size has equal probability of being selected.

A **Systematic Sample** is a sample collected by organizing the population into a list, randomly selecting a starting point, and sampling every n^{th} value until the sample size is reached.

A **Stratified Sample** is a sample collected by first grouping the population into groups called strata and then selecting a random sample from each stratum.

A **Cluster Sample** is a sample collected by first grouping the population into groups called clusters and then sampling all individuals in one or more clusters that have been randomly selected.

A **Convenience Sample** is a sample collected at the researcher's convenience.

Chapter 2 Exercises

1. Which types of graphs are used for quantitative data? Select all that apply. Answers a, c, d
   a) Ogive
   b) Pie Chart
   c) Histogram
   d) Stem-and-Leaf Plot
   e) Bar Graph

   Both **pie charts** and **bar graphs** are used to represent data that has been separated into categories. When data are separated into categories, it is qualitative data.

   Ogives, **histograms**, and **stem-and-leaf plots** are used to represent data that take on numeric values. When data are made up of numeric values, it is quantitative data.

3. The bars for a histogram should always touch, true or false? True

5. An instructor had the following grades recorded for an exam.

96	66	65	82	85
82	87	76	80	85
83	69	79	70	83
63	81	94	71	83
99	75	73	83	86

   a) Create a stem-and-leaf plot.

6	3 5 6 9
7	0 1 3 5 6 9
8	0 1 2 2 3 3 3 3 5 5 6 7
9	4 6 9

   b) Complete the following table.

Class	Frequency	Cumulative Frequency	Relative Frequency	Cumulative Relative Frequency
60-69	4	4	4/25 = 0.16	4/25 = 0.16
70-79	6	4+6 =10	6/25 = 0.24	10/25 = 0.4
80-89	12	10+12 = 22	12/25 = 0.48	22/25 = 0.88
90-99	3	22+3 = 25	3/25 = 0.12	25/25 = 1
Total	25		1	

   c) What should the relative frequencies always add up to? Answer = 1. Since relative frequencies can be converted into percentages, the total relative frequency should add up to 100%. Converting 100% back to decimal form gives us a total relative frequency of 1.

d) What should the last value always be in the cumulative frequency column? The sample size.
e) What is the frequency for students that were in the C range of 70-79? 6
f) What is the relative frequency for students that were in the C range of 70-79? 0.24
g) Which is the modal class? 80-89 is the most frequent
h) Which is the class has a relative frequency of 12%? 90-99
i) What is the cumulative frequency for students that were in the B range of 80-89? 0.88
j) Which class has a cumulative relative frequency of 40%? 70-79

7. The following table is from a sample of five hundred homes in Oregon asked the primary source of heating their residential home.

Type of Heat	Percent
Electricity	33
Heating Oil	4
Natural Gas	50
Firewood	8
Other	5

a) How many of the households heat their home with firewood? 40 households. The table above is giving a percentage for each household, 8% using Firewood. 8% of the 500 homes in the sample gives the following: $0.08 \cdot 500 = 40$
b) What percent of households heat their home with natural gas? 50%. Read this directly from the table.

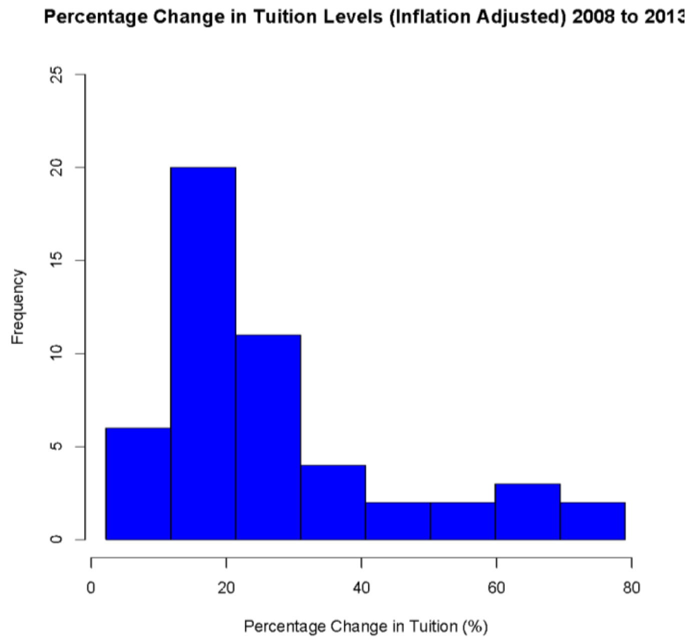
9. A sample of heights of 20 people in cm is recorded below. Make a stem-and-leaf plot.

Height (cm)					
167	201	170	185	175	162
182	186	172	173	188	154
185	178	177	184	178	165
169	171	185	178	175	176

15	4
16	2 5 7 9
17	0 1 2 3 5 5 6 7 8 8 8
18	2 4 5 5 5 6 8
19	
20	1

Note: The smallest given data value is 154 and the largest is 201. The stems for each of these are 15 and 20 respectively, being that the stem includes all digits in the data value except the right most digit. The stems on the plot need to include every whole number from 15 to 20 (including 19, as you see above, despite there being no data values with a stem of 19).

11. The following data represents the percent change in tuition levels at public, four-year colleges (inflation adjusted) from 2008 to 2013 (Weissmann, 2013). Below is the frequency distribution and histogram.



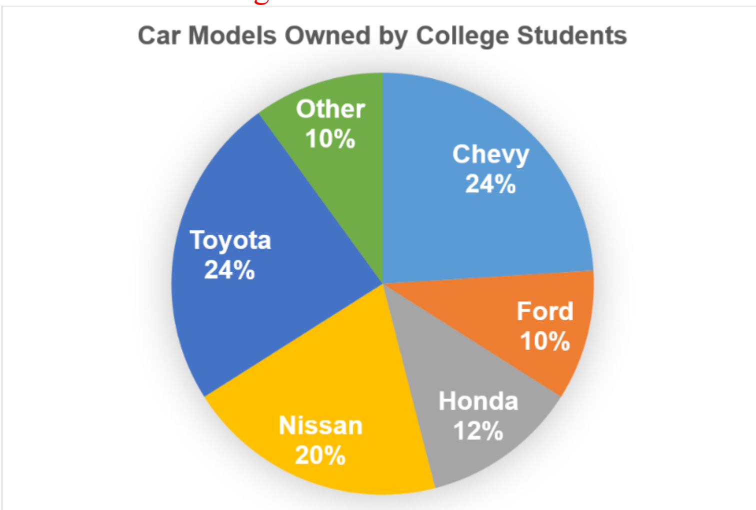
Class Limits	Class Midpoint	Frequency	Relative Frequency
2.2 – 11.7	6.95	6	0.12
11.8 – 21.3	16.55	20	0.40
21.4 – 30.9	26.15	11	0.22
31.0 – 40.5	35.75	4	0.08
40.6 – 50.1	45.35	2	0.04
50.2 – 59.7	54.95	2	0.04
59.8 – 69.3	64.55	3	0.06
69.4 – 78.9	74.15	2	0.04

a) How many colleges were sampled? 50. Find the total number sampled by adding up all the values in the frequency column of the frequency distribution.
$6 + 20 + 11 + 4 + 2 + 2 + 3 + 2 = 50$

b) What was the approximate value of the highest change in tuition? 78. We don't have the original data to know the exact highest value, but we can approximate the highest value from both the histogram and the classes in the frequency distribution. The right-most value on the x-axis of the histogram is just under 80, and the last class of the frequency distribution is 69.4-78.9. Given these two pieces of information, the answer of 78 is the best approximation for the highest change in tuition.

c) What was the approximate value of the most frequent change in tuition? $\frac{(11.8+21.3)}{2} = 16.55$
While we don't have the original data to know that answer for sure, we do know that the most frequent class is 11.8-21.3 by observing that the frequency for that class is 20 (the highest frequency on the table). To approximate the most frequent change in tuition, we simply take the class midpoint of that class.

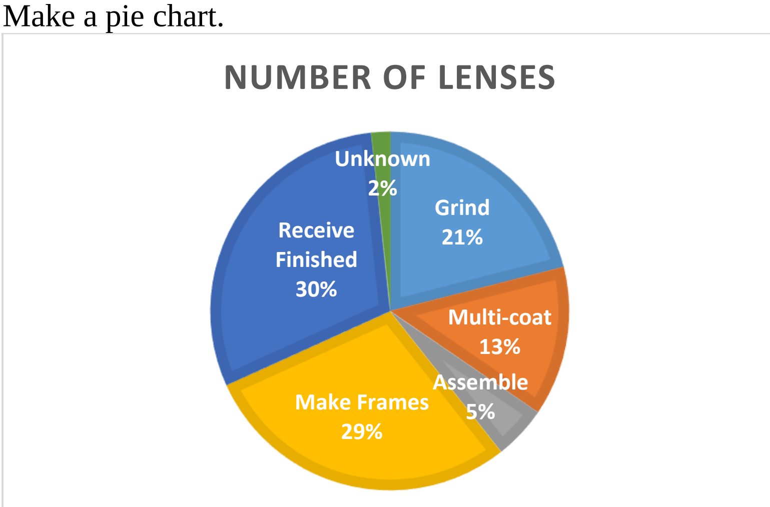
13. The following graph represents a random sample of car models driven by college students. What percent of college students drove a Nissan? 20%. The sector of the pie graph that is labeled "Nissan" is given to be 20%.



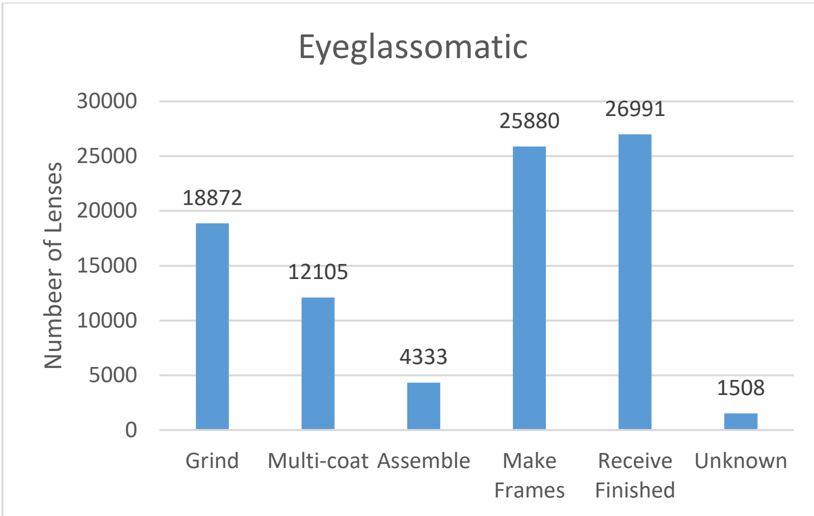
15. Eyeglassomatic manufactures eyeglasses for different retailers. The number of lenses for different activities is in table.

Activity	Grind	Multi-coat	Assemble	Make Frames	Receive Finished	Unknown
Number of lenses	18,872	12,105	4,333	25,880	26,991	1,508

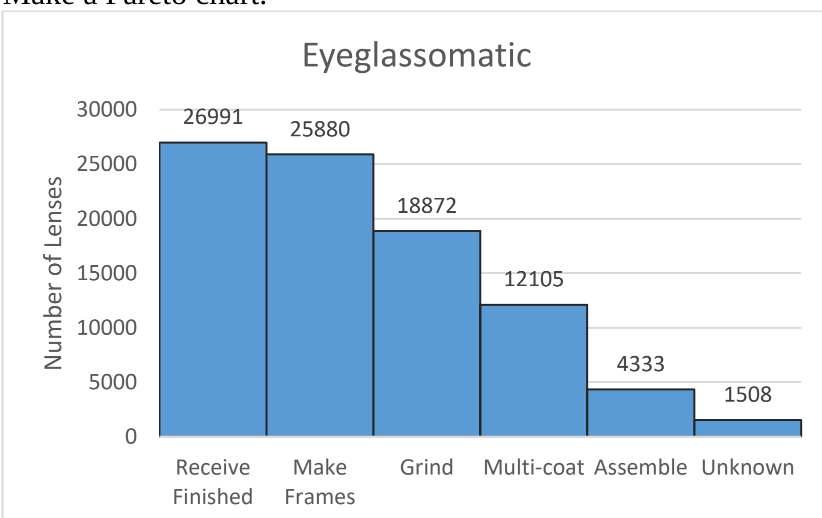
a) Make a pie chart.



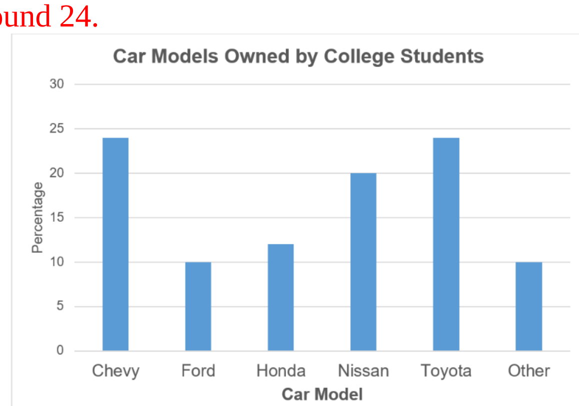
b) Make a bar chart.



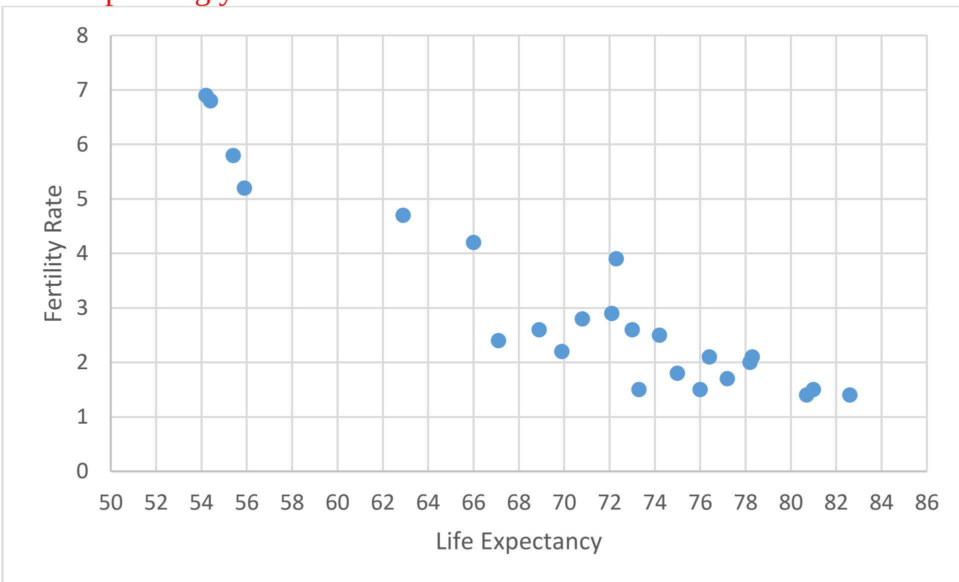
c) Make a Pareto chart.



17. The following graph represents a random sample of car models driven by college students. What was the most common car model? Chevy & Toyota – they each have a frequency of around 24.

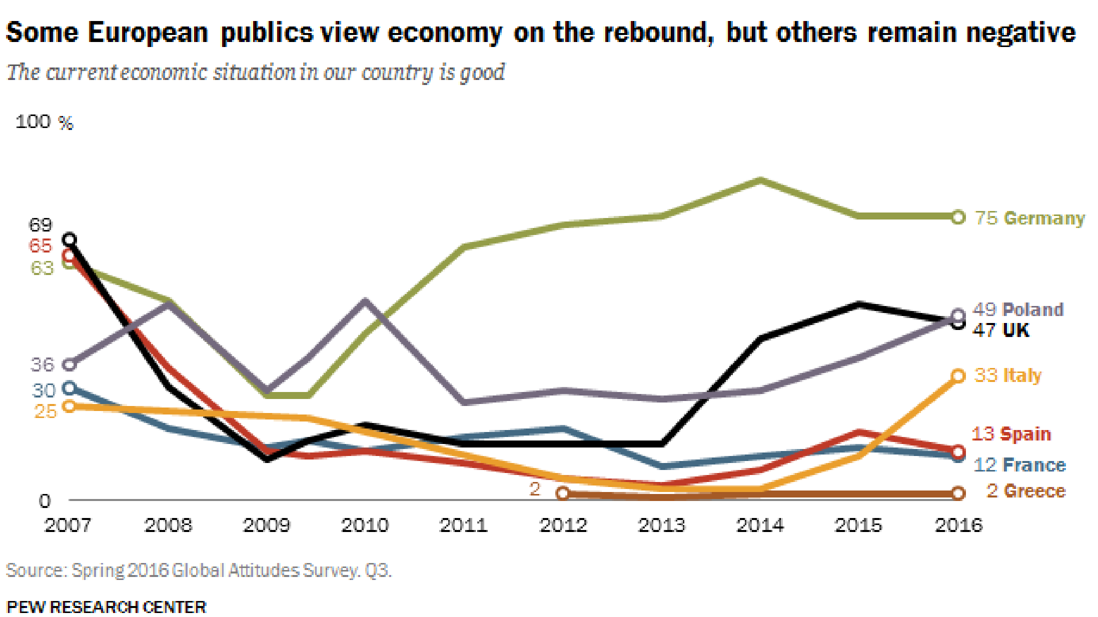


19. A scatter plot for a random sample of 24 countries shows the average life expectancy and the average number of births per woman (fertility rate). What is the approximate fertility rate for a country that has a life expectancy of 76 years? (2013, October 14). 1.5. The x-value of 76 has a corresponding y-value of 1.5.



Retrieved from http://data.worldbank.org/indicator/SP.DYN.TFRT.IN

21. A survey by the Pew Research Center, conducted in 16 countries among 20,132 respondents from April 4 to May 29, 2016, before the United Kingdom's so-called Brexit referendum to exit the EU. The following is a time series graph for the proportion of survey respondents by country that responded that the current economic situation is their country was good.

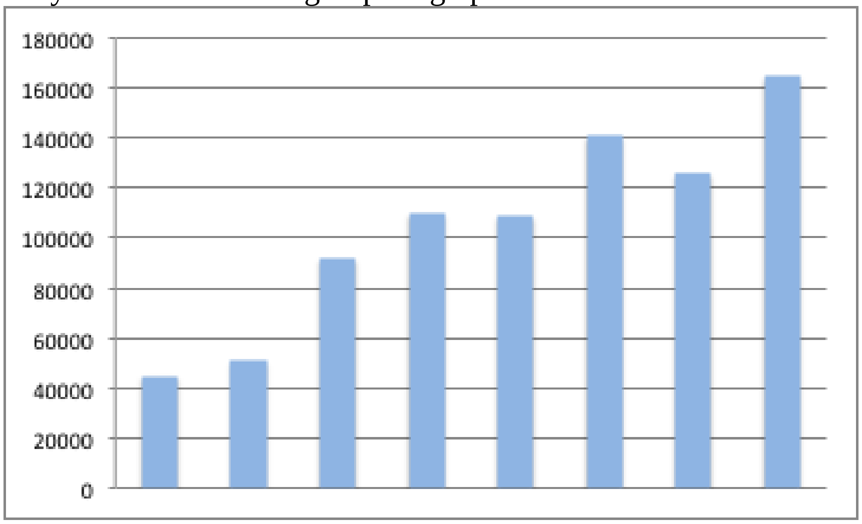


http://www.pewglobal.org/2016/08/09/views-on-national-economies-mixed-as-many-countries-continue-to-struggle/

a) Which country had the most favorable outlook of their country's economic situation in 2010? Poland, because in 2010 the highest line is the purple line, which represents Poland.

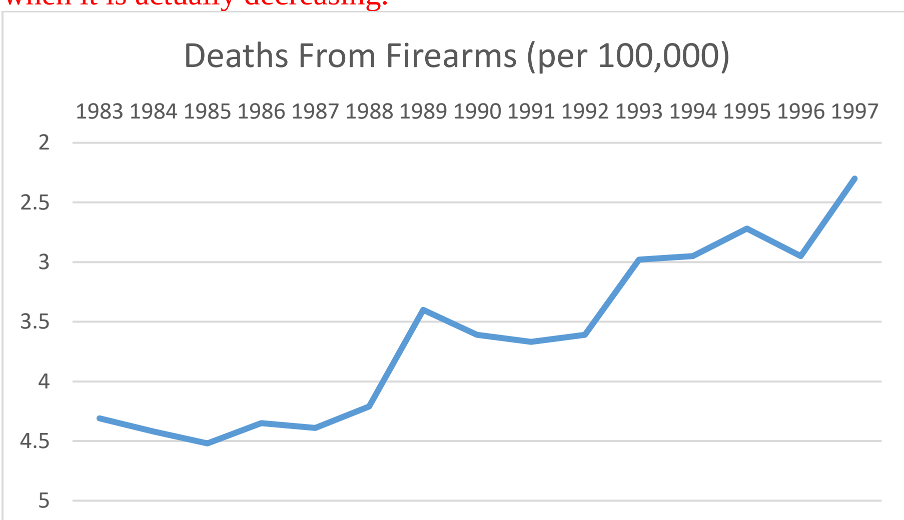
b) Which country had the least favorable outlook of their country's economic situation in 2016? Greece, because in 2016 the lowest line is the brown line, which represents Greece.

23. Why is this a misleading or poor graph?



There are no labels for both axis or categories.

25. The Australian Institute of Criminology gathered data on the number of deaths (per 100,000 people) due to firearms during the period 1983 to 1997 (2013, September 26). Why is this a misleading or poor graph? The vertical axis is reversed, making the graph appear to increase when it is actually decreasing.



Retrieved from http://www.statsci.org/data/oz/firearms.html.

Chapter 3 Exercises

1. A sample of 8 cats found the following weights in kg. Compute the mean, median and mode.

4.0	4.1	3.2	4.0	3.8	3.6	3.7	3.4

a) Compute the mode. Mode = 4.0 kg, because 4.0 occurs the most in the list of data.

b) Compute the median. Median = 3.75 kg, because 3.75 is the average of the middle two values in the sorted data.

Sorted list: 3.2 3.4 3.6 **3.7** **3.8** 4.0 4.0 4.1

$$\frac{3.7+3.8}{2} = 3.75$$

c) Compute the mean. $\bar{x} = \frac{4.0+4.1+3.2+4.0+3.8+3.6+3.7+3.4}{8} = 3.725\ kg$

3. The lengths (in kilometers) of rivers on the South Island of New Zealand that flow to the Tasman Sea are listed below.

River	Length (km)	River	Length (km)
Hollyford	76	Waimea	48
Cascade	64	Motueka	108
Arawhata	68	Takaka	72
Haast	64	Aorere	72
Karangarua	37	Heaphy	35
Cook	32	Karamea	80
Waiho	32	Mokihinui	56
Whataroa	51	Buller	177
Wanganui	56	Grey	121
Waitaha	40	Taramakau	80
Hokitika	64	Arahura	56

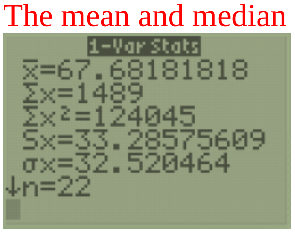
Data from http://www.statsci.org/data/oz/nzrivers.html

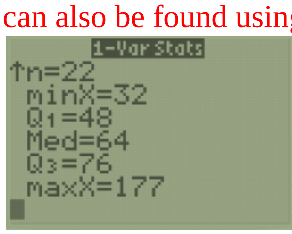
a) Compute the mode. 56 km & 64 km – both 56 and 64 occur three times, so there are two modes in this case.

b) Compute the median. 64 km – find the median by sorting the data from lowest to highest and then taking the middle value. Since there are *two* middle values in this case, take the average of the two $\frac{64+64}{2} = 64$.

c) Compute the mean. 67.6818 km – find the mean by adding up all the data values and dividing the result by the total number of data values.

The mean and median can also be found using 1-Var Stats in your calculator.





5. A university assigns letter grades with the following 4-point scale: A = 4.00, A– = 3.67, B+ = 3.33, B = 3.00, B– = 2.67, C+ = 2.33, C = 2.00, C– = 1.67, D+ = 1.33, D = 1.00, D– = 0.67, F = 0.00. Calculate the grade point average (GPA) for a student who took in one term a 3-credit biology course and received a C+, a 1-credit lab course and received a B, a 4-credit engineering course and received an A- and a 4-credit chemistry course and received a C+.
   Find the weighted mean by multiplying each score by the corresponding weights and adding up the results for the numerator. The denominator will be the sum of the weights.

$$\frac{(3 \cdot 2.33 + 1 \cdot 3 + 4 \cdot 3.67 + 4 \cdot 2.33)}{3 + 1 + 4 + 4} = 2.833$$

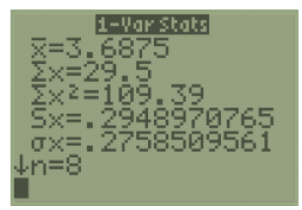
7. A statistics class has the following activities and weights for determining a grade in the course: test 1 worth 15% of the grade, test 2 worth 15% of the grade, test 3 worth 15% of the grade, homework worth 10% of the grade, semester project worth 20% of the grade, and the final exam worth 25% of the grade. If a student receives an 85 on test 1, a 76 on test 2, an 83 on test 3, a 74 on the homework, a 65 on the project, and a 61 on the final, what grade did the student earn in the course? All the assignments were out of 100 points.
   Find the weighted mean by multiplying each score by the corresponding weights and adding up the results for the numerator. The denominator will be the sum of the weights.

$$\frac{15 \cdot 85 + 15 \cdot 76 + 15 \cdot 83 + 10 \cdot 74 + 20 \cdot 65 + 25 \cdot 61}{15 + 15 + 15 + 10 + 20 + 25} = 72.25$$

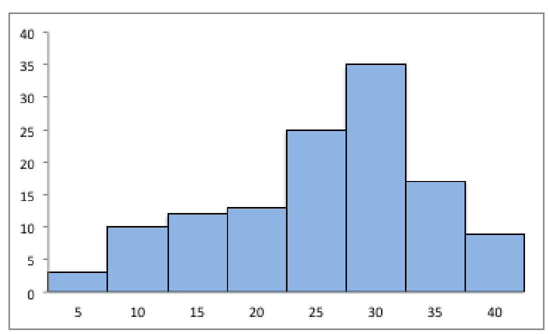
9. A sample of 8 cats found the following weights in kg.

3.7	4.1	3.2	4.0	3.8	3.6	3.7	3.4

   a) Compute the range. Max – Min = 4.1 – 3.2 = 0.9
   b) Compute the variance. $s^2 = 0.2949^2 = 0.087$
   c) Compute the standard deviation. s = 0.2949



11. The following is a histogram of quiz grades.



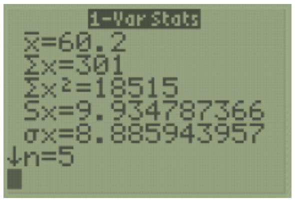
   a) What is the shape of the distribution? Negatively skewed, since the peak of the graph is on the right side and there is a tail to the left.
   b) Which is higher, the mean or the median? The mean is pulled in the direction of the tail, so the median would be higher.

13. Suppose that a manager wants to test two new training programs. The manager randomly selects 5 people for each training type and measures the time it takes to complete a task after the training. The times for both trainings are in table below. Which training method is more variable?

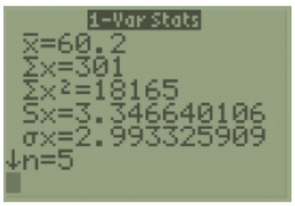
Training 1	56	75	48	63	59
Training 2	60	58	66	59	58

Since the scale and units are the same, we can compare standard deviations.

$$s_1 = \sqrt{\frac{(56-60.2)^2+(75-60.2)^2+(48-60.2)^2+(63-60.2)^2+(59-60.2)^2}{5-1}} = 9.9348$$



$$s_2 = \sqrt{\frac{(60-60.2)^2+(58-60.2)^2+(66-60.2)^2+(59-60.2)^2+(58-60.2)^2}{5-1}} = 3.3466$$



Training type 1 is more variable since it has a larger standard deviation.

15. Here are pulse rates before and after exercise. Which group has the larger range?

Before	**Pulse Rates**	**After**
9 8 8 7 6 5 2	6	
9 8 8 8 6 5 5 5 1 1 0 0	7	
8 8 7 5 4 2	8	5 6 6 7 8 9
4 0	9	0 1 1 2 3 4 5 5 6 8
4	10	0 1 4 6 7
	11	6 7
	12	4 5 7

Find the range of each using $Max - Min$.
Before range = 104 – 62 = 42, after range = 127 – 85 = 42. Both groups have the same range.

17. The following is a sample of quiz scores.

Score	17	44.5	16.1	37.2	42.8	37.5	19.5	28.2

a) Compute $\bar{x}$. 30.35
b) Compute s^2. 136.7
c) Compute the median. 32.7
d) Compute the coefficient of variation. 32.7/2035 = 38.52%
e) Compute the range. Max – Min = 28.4

19. The following is the height and weight of a random sample of baseball players.

Height (inches)	Weight (pounds)
76	212
76	224
72	180
74	210
75	215
71	200
77	235
78	235
77	194
76	185
72	180
72	170
75	220
74	228
73	210
72	180
70	185
73	190
71	186
74	200
74	200
75	210
78	240
72	208
75	180

a) Compute the coefficient of variation for both height and weight.

	Height (inches)	*Weight (pounds)*
Mean	74.08	203.08
Standard Deviation	2.253146	19.98733
Coefficient of Variation	$\frac{2.253146}{74.08} \cdot 100\% = 3.04\%$	$\frac{19.98733}{203.08} \cdot 100\% = 9.84\%$

b) Is there more variation in height or weight? Weight, because it has a higher coefficient of variation.

21. The length of a human pregnancy is normally distributed with a mean of 272 days with a standard deviation of 9.1 days. William Hunnicut was born in Portland, Oregon, at just 181 days into his gestation. What is the z-score for William Hunnicut's gestation?
Retrieved from: http://digitalcommons.georgefox.edu/cgi/viewcontent.cgi?article=1149&context=gfc_life

$z = \frac{x-\bar{x}}{s} = \frac{181-272}{9.1} = -10$, this is 10 standard deviations below average.

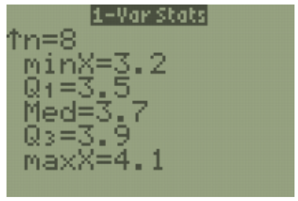
23. The average time to run the Pikes Peak Marathon 2017 was 7.44 hours with a standard deviation of 1.34 hours. Rémi Bonnet won the Pikes Peak Marathon with a run time of 3.62 hours. Retrieved from: http://pikespeakmarathon.org/results/ppm/2017/.
The Tevis Cup 100-mile one day horse race for 2017 had an average finish time of 20.38 hours with a standard deviation of 1.77 hours. Tennessee Lane won the 2017 Tevis cup in a ride time of 14.75 hours. Retrieved from: https://aerc.org/rpts/RideResults.aspx.

a) Compute the z-score for Rémi Bonnet's time. $z = \frac{3.62-7.44}{1.34} = -2.8507$
b) Compute the z-score for Tennessee Lane's time. $z = \frac{14.75-20.38}{1.77} = -3.1808$
c) Which competitor did better compared to their respective events? "Better" for race times would be the smaller of the two z-scores, so Tennessee Lane did better.

25. A sample of 8 cats found the following weights in kg. Compute the 5-number summary.

3.7	4.1	3.2	4.0	3.8	3.6	3.7	3.4

Excel	=QUARTILE.EXC	=QUARTILE.INC
Minimum	3.2	3.2
Q1	3.45	3.55
Q2	3.7	3.7
Q3	3.95	3.85
Maximum	4.1	4.1



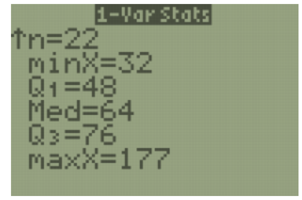
27. The lengths (in kilometers) of rivers on the South Island of New Zealand that flow to the Tasman Sea are listed below.

River	Length (km)	River	Length (km)
Hollyford	76	Waimea	48
Cascade	64	Motueka	108
Arawhata	68	Takaka	72
Haast	64	Aorere	72
Karangarua	37	Heaphy	35
Cook	32	Karamea	80
Waiho	32	Mokihinui	56
Whataroa	51	Buller	177
Wanganui	56	Grey	121
Waitaha	40	Taramakau	80
Hokitika	64	Arahura	56

Data from http://www.statsci.org/data/oz/nzrivers.html

a) Compute the 5-number summary.
Note: The values you find for the quartiles may differ depending on what form of technology you use to calculate them.

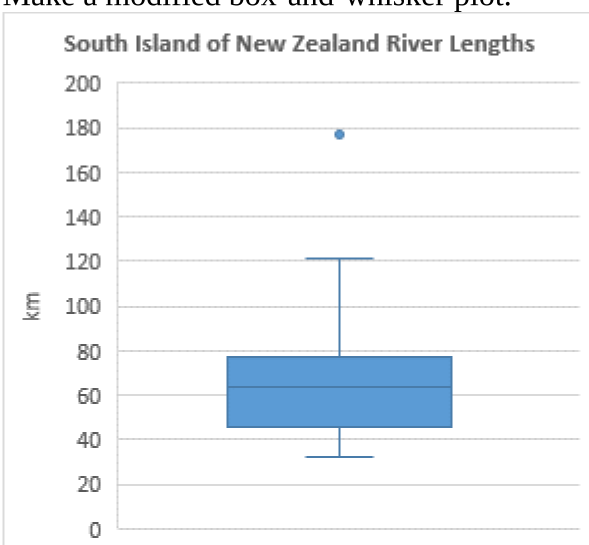
	Answers	Excel Formulas
Min	32	=MIN
Q_1	46 (Excel) or 48 (TI)	=QUARTILE.EXC
Q_2	64	=MEDIAN
Q_3	77 (Excel) or 76 (TI)	=QUARTILE.EXC
Max	177	=MAX

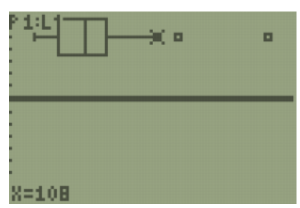


b) Compute the lower and upper limits and any outlier(s) if any exist.
Excel: IQR = $Q_3 - Q_1$ = 77 – 46 = 31, Lower Limit = Q_1 – 1.5*IQR = 46 – 1.5*31 = -0.5, Upper Limit = Q3 + 1.5*IQR = 77 + 1.5*31 = 123.5. Any data value outside the limits (-0.5, 123.5) is an outlier. Outlier = 177. Note the whiskers go out to the data values 32 and 121, not the limits.

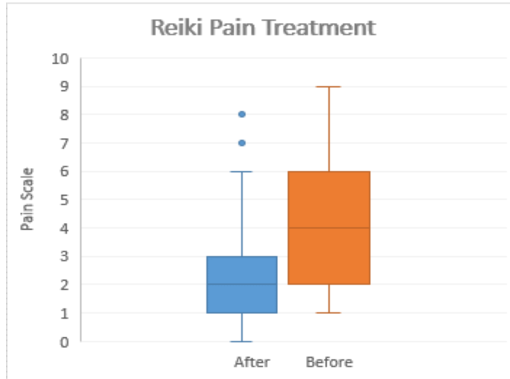
TI-Calculator: IQR = $Q_3 - Q_1$ = 76 – 48 = 28, Lower Limit = Q_1 – 1.5*IQR = 48 – 1.5*28 = 6, Upper Limit = Q3 + 1.5*IQR = 76 + 1.5*28 = 118. Any data value outside the limits (-0.5, 123.5) is an outlier. Outliers = 121 and 177. Note the whiskers go out to the data values 32 and 108, not the limits.

c) Make a modified box-and-whisker plot.





29. To determine if Reiki is an effective method for treating pain, a pilot study was carried out where a certified second-degree Reiki therapist provided treatment on volunteers. Pain was measured using a visual analogue scale (VAS) immediately before and after the Reiki treatment (Olson & Hanson, 1997). Higher numbers mean the patients had more pain.

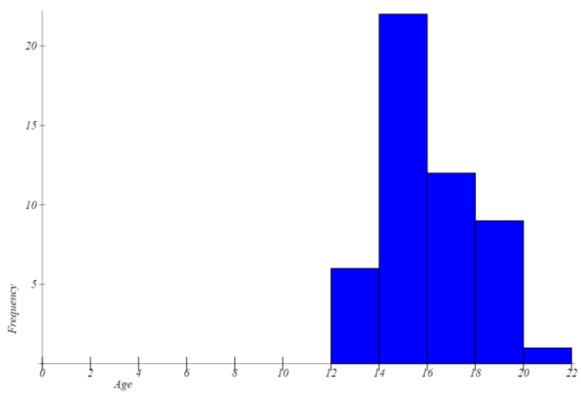


a) Use the box-and-whiskers plots to determine the IQR for the before treatment measurements. IQR = $Q_3 - Q_1 = 6 - 2 = 4$

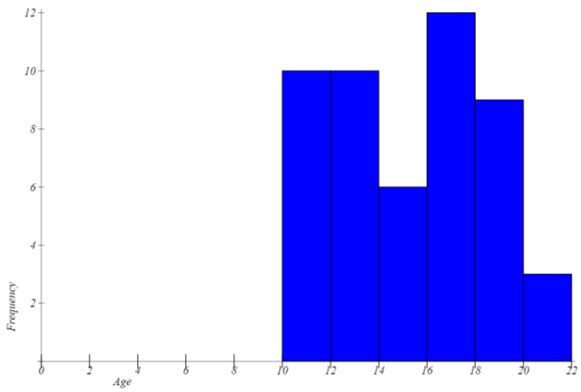
b) Use the box-and-whiskers plots of the before and after VAS ratings to determine if the Reiki method was effective in reducing pain. Yes, the treatment was effective since all 3 quartiles for the after treatment measurements were smaller than the before treatment measurements.

31. Match the correct descriptive statistics to the letter of the corresponding histogram and boxplot. Choose the correct letter for the corresponding histogram and Roman numeral for the corresponding boxplot. You should only use the visual representation, the definition of standard deviation and measures of central tendency to match the graphs with their respective descriptive statistics.

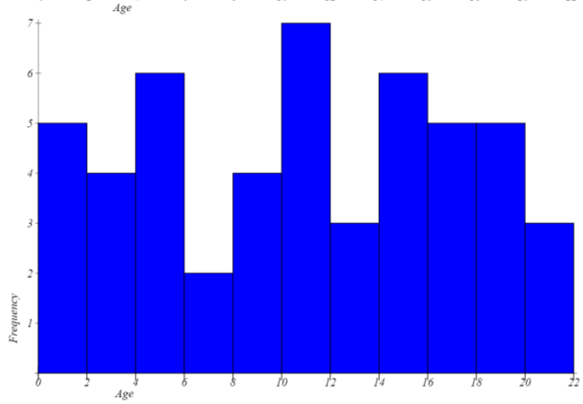
	Mean	Median	Standard Deviation	Histogram Letter	Boxplot Number
1	10.4	11	6.2		
2	16	15.8	1.9		
3	14.8	15	3.1		

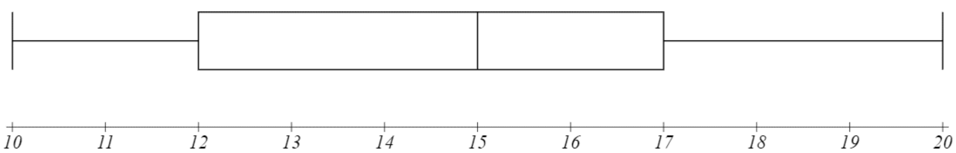
a) 

b)

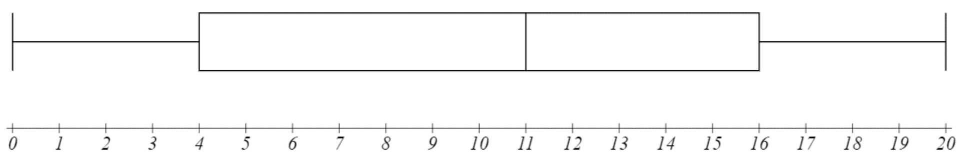


c)

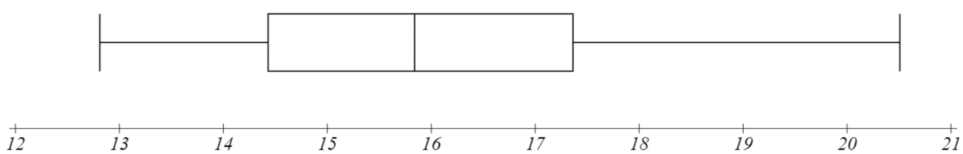




i.



ii.



iii.

	Mean	Median	Standard Deviation	Histogram Letter	Boxplot Number
1	10.4	11	6.2	c	ii
2	16	15.8	1.9	a	iii
3	14.8	15	3.1	b	i

33. The length of a human pregnancy is bell-shaped with a mean of 272 days with a standard deviation of 9 days (Bhat & Kushtagi, 2006). Find the percentage of pregnancies that last between 245 and 299 days.
$z = \frac{245-272}{9} = -3, z = \frac{299-272}{9} = 3$
Within 3 standard deviations from the mean according to the Empirical Rule would have approximately 99.7%.

35. The size of fish is very important to commercial fishing. A study conducted in 2012 found the length of Atlantic cod caught in nets in Karlskrona to have a mean of 49.9 cm and a standard deviation of 3.74 cm (Ovegard, Berndt & Lunneryd, 2012).

a) According to Chebyshev's Inequality, at least what percent of Atlantic cod should be between 44.29 and 55.51 cm? $z = \frac{44.29-49.9}{3.74} = -1.5, z = \frac{55.51-49.9}{3.74} = 1.5$

Within 1.5 standard deviations from the mean according to Chebyshev's Inequality would give at least $1 - \frac{1}{1.5^2} = 0.5556$ or 55.56%.

b) Assume the length of Atlantic cod is bell-shaped. Approximately what percent of Atlantic cod are between 46.16 and 53.64 cm? $z = \frac{46.16-49.9}{3.74} = -1, z = \frac{53.64-49.9}{3.74} = 1$

Within 1 standard deviation from the mean according to the Empirical Rule would have approximately 68%.

c) Assume the length of Atlantic cod is bell-shaped. Approximately what percent of Atlantic cod are between 42.42 and 57.38 cm? $z = \frac{42.42-49.9}{3.74} = -2, z = \frac{57.38-49.9}{3.74} = 2$

Within 2 standard deviations from the mean according to the Empirical Rule would have approximately 95%.

37. In a mid-size company, the distribution of the number of phone calls answered each day by each of the 12 employees is bell-shaped and has a mean of 59 and a standard deviation of 10. Using the empirical rule, what is the approximate percentage of daily phone calls numbering between 29 and 89? 99.7%

39. A company has a policy of retiring company cars; this policy looks at number of miles driven, purpose of trips, style of car and other features. The distribution of the number of months in service for the fleet of cars is bell-shaped and has a mean of 42 months and a standard deviation of 3 months. Using the Empirical Rule, what is the approximate percentage of cars that remain in service between 48 and 51 months? 2.35%

Chapter 4 Exercises

1. The number of M&M candies for each color found in a case were recorded in the table below.

Blue	Brown	Green	Orange	Red	Yellow	Total
481	371	483	544	372	369	2,620

What is the probability of selecting a red M&M? 372/2620 = 0.1420

3. An experiment is to flip a fair coin three times. What is the probability of getting exactly two heads? There are 3 outcomes with exactly two heads and a total of 8 outcomes in the sample space P(2 H) = 3/8 = 0.375

5. A raffle sells 1000 tickets for $35 each to win a new car. What is the probability of winning the car? There is only one winning ticket, 1/1000 = 0.001.

7. Compute the probability of rolling a sum of two dice that is a 7 or a 12. There are 7 ways to get a sum of 7 or 12, P(7 or 12) = 7/36 = 0.1944.

+	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

9. The probability that a consumer entering a retail outlet for microcomputers and software packages will buy a computer of a certain type is 0.15. The probability that the consumer will buy a particular software package is 0.10. There is a 0.05 probability that the consumer will buy both the computer and the software package. What is the probability that the consumer will buy the computer or the software package?
P(Computer or SP) = P(Computer) + P(SP) – P(Computer and SP) = 0.15 + 0.10 – 0.05 = 0.2.

11. Giving a test to a group of students, the grades and if they were business majors are summarized below. One student is chosen at random. Give your answer as a decimal out to at least 4 places.

	A	B	C	Total
Business Majors	4	5	13	22
Non-business Majors	18	10	19	47
Total	22	15	32	69

a) Compute the probability that the student was a non-business major or got a grade of C. P(NB or C) = P(NB) + P(C) – P(NB and C) = 47/69 + 32/69 – 19/69 = 60/69 = 0.8696
b) Compute the probability that the student was a non-business major and got a grade of C. P(NB and C) = 19/69 = 0.2754

c) Compute the probability that the student was a non-business major given they got a grade of C. P(NB|C) = P(NB and C)/P(C) = 19/32 = 0.5938
d) Compute the probability that the student did not get a B grade. $P(B^C)$ = 1 – P(B) = 1 – 15/69 = 54/69 = 0.7826
e) Compute P(B ∪ Business Major). P(B) + P(Bus) – P(B and Bus) = 15/69 + 22/69 – 5/69 = 32/69 = 0.4638
f) Compute P(C | Business Major). P(C and Bus)/P(Bus) = 13/22 = 0.5909

13. Your favorite basketball player is an 81% free throw shooter. Find the probability that he does not make their next free throw shot. 1 – 0.81 = 0.19 or 19%

15. The smallpox data set provides a sample of 6,224 individuals from the year 1721 who were exposed to smallpox in Boston.

	Inoculated	Not Inoculated	Total
Lived	238	5136	5374
Died	6	844	850
Total	244	5980	6224

Fenner F. 1988. Smallpox and Its Eradication (History of International Public Health, No. 6). Geneva: World Health Organization. ISBN 92-4-156110-6.

a) Compute the relative frequencies.

	Inoculated	Not Inoculated	Total
Lived	0.0382	0.8252	0.8634
Died	0.0010	0.1356	0.1366
Total	0.0392	0.9608	1

b) Compute the probability that a person was inoculated. 0.0392
c) Compute the probability that a person lived. 0.8634
d) Compute the probability that a person died or was inoculated. 0.1366 + 0.0392 – 0.0010 = 0.1748
e) Compute the probability that a person died if they were inoculated. P(Died | Inoculated) = P(Died and Inoculated)/P(Inoculated) = 0.001/0.0392 = 0.026
f) Given that a person was not inoculated, what is the probability that they died? P(Died | Not Inoculated) = P(Died and Not Inoculated)/P(Not Inoculated) = 0.1356/0.9608 = 0.141

17. A store purchases baseball hats from three different manufacturers. In manufacturer A's box there are 12 blue hats, 6 red hats, and 6 green hats. In manufacturer B's box there are 10 blue hats, 10 red hats, and 4 green hats. In manufacturer C's box, there are 8 blue hats, 8 red hats, and 8 green hats. A hat is randomly selected. Given that the hat selected is green, what is the probability that it came from manufacturer B's box? Hint: Make a table with the colors as the columns and the manufacturers as the rows.
Make a table:

	Blue	Red	Green	Total
A	12	6	6	24
B	10	10	4	24
C	8	8	8	24
Total	30	24	18	72

Find P(B | Green) = $\frac{P(B \cap Green)}{P(Green)} = \frac{4}{18}$ = 0.2222

19. The probability of stock A rising is 0.3; and of stock B rising is 0.4. What is the probability that neither of the stocks rise, assuming that these two stocks are independent? Since A and B are independent then P(A and B) = P(A)*P(B) = 0.3*0.4 = 0.12. The probability that either stocks rise is P(A or B) = P(A) + P(B) – P(A and B) = 0.3 + 0.4 – 0.12 = 0.58. The probability of neither would be the complement to either which is 1 – 0.58 = 0.42.

21. How many different phone numbers are possible in the area code 503, if the first number cannot start with a 0 or 1? 8*10*10*10*10*10*10 = 8,000,000

23. The California license plate has one number followed by three letters followed by three numbers. How many different license plates are possible? 10*24*24*24*10*10*10 = 138,240,000

25. The PSU's Mixed Me club has 30 members. You need to pick a president, treasurer, and secretary from the 30. How many different ways can you do this? ${}_{30}P_3$ = 24,360

27. A baseball team has a 20-person roster. A batting order has nine people. How many different batting orders are there? ${}_{20}P_9$ = 60,949,324,800

29. A computer generates a random password for your account (the password is not case sensitive). The password must consist of 8 characters, each of which can be any letter or number. How many different passwords could be generated? 36^8 = 2,821,109,907,456

31. A typical PSU locker is opened with correct sequence of three numbers between 0 and 49 inclusive. A number can be used more than once, for example, 8-8-8 is valid. How many possible locker combinations are there? 50*50*50 = 125,000

Chapter 5 Exercises

1. Determine if the following tables are valid discrete probability distributions. If they are not state why.

   a) Yes, since all the probabilities are between 0 and 1 and the probabilities add up to one.

x	–5	–2.5	0	2.5	5
P(X = x)	0.15	0.25	0.32	0.18	0.1

   b) No, since the probabilities do not add up to 1.

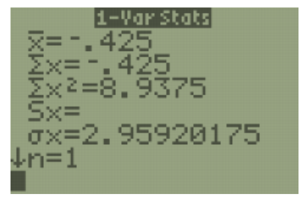
x	0	1	2	3	4
P(X = x)	0.111	0.214	0.312	0.163	0.159

   c) No, since not all the probabilities are between 0 and 1.

x	0	1	2	3	4
P(X = x)	0.2	–0.3	0.5	0.4	0.2

3. The following discrete probability distribution represents the amount of money won for a raffle game.

x	–5	–2.5	0	2.5	5
P(X = x)	0.15	0.25	0.32	0.18	0.1



   a) Compute μ. $\mu = \sum x \cdot P(x) = (-5) \cdot 0.15 + (-2.5) \cdot 0.25 + 0 \cdot 0.32 + 2.5 \cdot 0.18 + 5 \cdot 0.1$ = –$0.425

   b) Compute σ.
   $\sigma = \sqrt{\left(\sum x^2 \cdot P(x)\right) - \mu^2} = \sqrt{((-5)^2 \cdot 0.15 + (-2.5)^2 \cdot 0.25 + 0^2 \cdot 0.32 + 2.5^2 \cdot 0.18 + 5^2 \cdot 0.1) - (-.425)^2} = \sqrt{8.756875}$ = $2.9592

5. The bookstore also offers a chemistry textbook for $159 and a book supplement for $41. From experience, they know about 25% of chemistry students just buy the textbook while 60% buy both the textbook and supplement, the remaining 15% of students do not buy either book. Find the standard deviation of the bookstore revenue.
   $\mu = \sum x \cdot P(x) = 159 \cdot 0.25 + 200 \cdot 0.6 + 0 \cdot 0.15$ = $159.75
   $\sigma = \sqrt{\left(\sum x^2 \cdot P(x)\right) - \mu^2} = \sqrt{(159^2 \cdot 0.25 + 200^2 \cdot 0.6 + 0^2 \cdot 0.15) - (159.75)^2} = \sqrt{4800.1875}$ = $69.283

7. An LG Dishwasher, which costs $1000, has a 24% chance of needing to be replaced in the first 2 years of purchase. If the company has to replace the dishwasher within the two-year extended warranty, it will cost the company $112.10 to replace the dishwasher.

   a) Fill out the probability distribution for the value of the extended warranty from the perspective of the company.

x	-112.1	887.9
P(X = x)	0.76	0.24

   b) What is the expected value of the extended warranty? $\mu = \sum x \cdot P(x) = (-112.1) \cdot 0.76 + 887.9 \cdot 0.24$ = $127.90

c) Write a sentence interpreting the expected value of the warranty. For many of these extended warranties bought by customers, they can expect to gain 127.9 dollars per warranty on average.

9. The following table represents the probability of the number of pets owned by a college student.

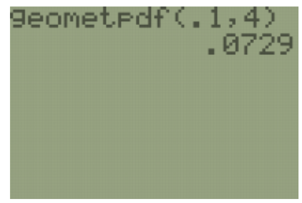
x	0	1	2	3
P(X = x)	0.46	0.35	0.12	0.07

a) Is this a valid discrete probability distribution? Explain your answer. Yes, since $\Sigma P(x) = 1$ and $0 \leq P(x) \leq 1$

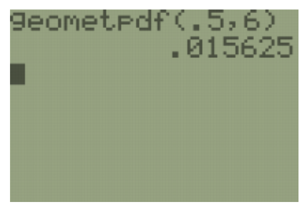
b) Find the mean number of pets owned. $\mu = \sum x \cdot P(x) = 0 \cdot 0.46 + 1 \cdot 0.35 + 2 \cdot 0.12 + 3 \cdot 0.07 = 0.8$

c) Find the standard deviation of the number of cars owned. $\sigma = \sqrt{(\sum x^2 \cdot P(x)) - \mu^2} = \sqrt{(0^2 \cdot 0.46 + 1^2 \cdot 0.35 + 2^2 \cdot 0.12 + 3^2 \cdot 0.07) - 0.8^2} = \sqrt{0.82} = 0.9055$

d) Find σ^2. 0.82



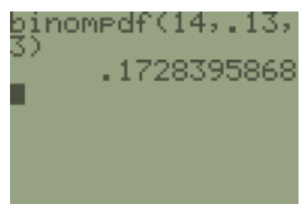
11. Approximately 10% of all people are left-handed. You randomly sample people until you get someone who is left-handed. What is the probability that the 4^{th} person selected will be the left-handed person? Geometric $P(X = x) = p \cdot q^{(x-1)}$, $p = 0.1$, $P(X = 4) = 0.1 \cdot 0.9^{(4-1)} = 0.0729$



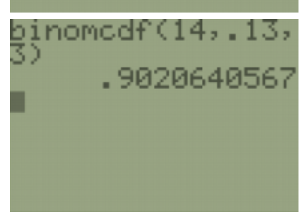
13. A fair coin is flipped until a head is shown. What is the probability that head shows on the 6^{th} flip? Geometric $p = 0.5$, $P(X = 6) = 0.5 \cdot 0.5^{(6-1)} = 0.0156$

15. Suppose a random variable, X, arises from a binomial experiment. If $n = 14$, and $p = 0.13$, find the following probabilities.

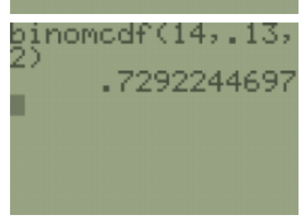
a) P(X = 3)
$P(X = x) = {}_nC_x \cdot p^x \cdot q^{(n-x)}$, binompdf(14, 0.13, 3), $P(X = 3) = {}_{14}C_3 \cdot 0.13^3 \cdot 0.87^{(14-3)} = 0.1728$



b) $P(X \leq 3)$
binomcdf(14, 0.13, 3), $P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = {}_{14}C_0 \cdot 0.13^0 \cdot 0.87^{(14-0)} + {}_{14}C_1 \cdot 0.13^1 \cdot 0.87^{(14-1)} + {}_{14}C_2 \cdot 0.13^2 \cdot 0.87^{(14-2)} + {}_{14}C_3 \cdot 0.13^3 \cdot 0.87^{(14-3)} = 0.142321 + 0.297729 + 0.289174 + 0.172840 = 0.9021$

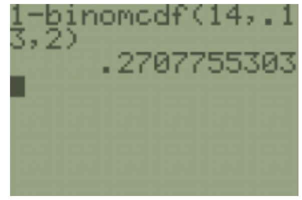


c) $P(X < 3)$
binomcdf(14,0.13,2), $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = {}_{14}C_0 \cdot 0.13^0 \cdot 0.87^{(14-0)} + {}_{14}C_1 \cdot 0.13^1 \cdot 0.87^{(14-1)} + {}_{14}C_2 \cdot 0.13^2 \cdot 0.87^{(14-2)} = 0.142321 + 0.297729 + 0.289174 = 0.7292$



d) P(X > 3)

P(X > 3) = 1 – P(X ≤ 3) = 1 – binomcdf(14,0.13,3) = 0.0979

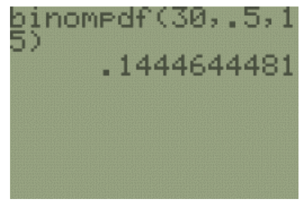


e) P(X ≥ 3)

P(X ≥ 3) = 1 – P(X ≤ 2) = 1 – binomcdf(14,0.13,2) = 0.2708

17. Suppose a random variable, X, arises from a binomial experiment. If n = 14, and p = 0.13, find the standard deviation. $\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{14 \cdot 0.13 \cdot 0.87} = 1.2583$

19. A fair coin is flipped 30 times. Binomial, n = 30, p = 0.5



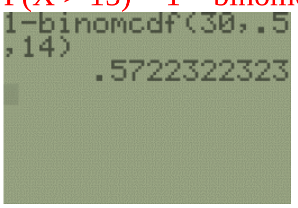
a) What is the probability of getting exactly 15 heads?

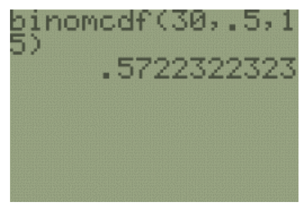
P(X = 15) = binompdf(30, 0.50, 15) = 0.1445

P(X = 15) = ${}_{30}C_{15} \cdot 0.5^{15} \cdot 0.5^{(30-15)}$ = 0.1445

b) What is the probability of getting 15 or more heads?

P(X > 15) = 1 – binomcdf(30, 0.50, 14) = 0.5722





c) What is the probability of getting at most 15 heads? P(X ≤ 15) = binomcdf(30, 0.50, 15) = 0.5722

d) How many times would you expect to get heads? $\mu = n \cdot p = 30 \cdot 0.5 = 15$

e) What is the standard deviation of the number of heads? $\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{30 \cdot 0.5 \cdot 0.5} = \sqrt{7.5} = 2.7386$

21. Approximately 8% of all people have blue eyes. Out of a random sample of 20 people, find the following. Binomial, n = 20, p = 0.08

a) What is the probability that 2 of them have blue eyes?

P(X = 2) = binompdf(20, 0.08, 2) = 0.2711

P(X = 2) = ${}_{20}C_{2} \cdot 0.08^{2} \cdot 0.92^{(20-2)}$ = 0.2711

b) What is the probability that at most 2 of them have blue eyes?

P(X ≤ 2) = binomcdf(20, 0.08, 2) = 0.7879

P(X ≤ 2) = P(X = 0) + P(X = 1) + P(X = 2) = ${}_{20}C_{0} \cdot 0.08^{0} \cdot 0.92^{(20-0)} + {}_{20}C_{1} \cdot 0.08^{1} \cdot 0.92^{(20-1)} + {}_{20}C_{2} \cdot 0.08^{2} \cdot 0.92^{(20-2)}$ = 0.7879

c) What is the probability that less than 2 of them have blue eyes?

P(X < 2) = binomcdf(20, 0.08, 1) = 0.5169

P(X < 2) = P(X = 0) + P(X = 1) = ${}_{20}C_{0} \cdot 0.08^{0} \cdot 0.92^{(20-0)} + {}_{20}C_{1} \cdot 0.08^{1} \cdot 0.92^{(20-1)}$ = 0.5169

d) What is the probability that at least 2 of them have blue eyes? $P(X \geq 2) = 1 -$ binomcdf(20,0.08,1) = 0.4831

e) What is the probability that more than 2 of them have blue eyes? $P(X > 2) = 1 -$ binomcdf(20,0.08,2) = 0.2121

f) Compute μ. $\mu = n \cdot p = 20 \cdot 0.08 = 1.6$

g) Compute σ. $\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{20 \cdot 0.08 \cdot 0.92} = \sqrt{1.472} = 1.2133$

h) Compute σ^2. $\sigma^2 = n \cdot p \cdot q = 20 \cdot 0.08 \cdot 0.92 = 1.472$

23. A local county has an unemployment rate of 7.3%. A random sample of 20 employable people are picked at random from the county and are asked if they are employed. The distribution is a binomial. Round answers to 4 decimal places. Binomial, n = 20, p = 0.073

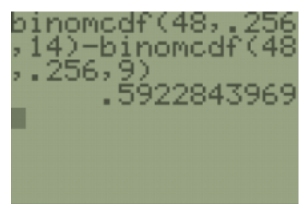
a) Find the probability that exactly 3 in the sample are unemployed. $P(X = 3) = {}_{20}C_3 \cdot 0.073^3 \cdot 0.927^{(20-3)} = 0.1222$

b) Find the probability that there are fewer than 4 in the sample are unemployed. $P(X < 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = {}_{20}C_0 \cdot 0.073^0 \cdot 0.927^{(20-0)} + {}_{20}C_1 \cdot 0.073^1 \cdot 0.927^{(20-1)} + {}_{20}C_2 \cdot 0.073^2 \cdot 0.927^{(20-2)} + {}_{20}C_3 \cdot 0.073^3 \cdot 0.927^{(20-3)} = 0.9464$

c) Find the probability that there are more than 2 in the sample are unemployed. $P(X > 2) = 1 - P(X \leq 2) = 1 - ({}_{20}C_0 \cdot 0.073^0 \cdot 0.927^{(20-0)} + {}_{20}C_1 \cdot 0.073^1 \cdot 0.927^{(20-1)} + {}_{20}C_2 \cdot 0.073^2 \cdot 0.927^{(20-2)}) = 0.1759$

d) Find the probability that there are at most 4 in the sample are unemployed. $P(X \leq 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = {}_{20}C_0 \cdot 0.073^0 \cdot 0.927^{(20-0)} + {}_{20}C_1 \cdot 0.073^1 \cdot 0.927^{(20-1)} + {}_{20}C_2 \cdot 0.073^2 \cdot 0.927^{(20-2)} + {}_{20}C_3 \cdot 0.073^3 \cdot 0.927^{(20-3)} + {}_{20}C_4 \cdot 0.073^4 \cdot 0.927^{(20-4)} = 0.9873$

25. You really struggle remembering to bring your lunch to work. Each day seems to be independent as to whether you remember to bring your lunch or not. The chance that you forget your lunch each day is 25.6%. Consider the next 48 days. Let X be the number of days that you forget your lunch out of the 48 days. Compute $P(10 \leq X \leq 14)$. Binomial, n = 48, p = 0.256, $P(10 \leq X \leq 14) = P(X = 10) + P(X = 11) + P(X = 12) + P(X = 13) + P(X = 14) = {}_{48}C_{10} \cdot 0.073^{10} \cdot 0.927^{(20-10)} + {}_{20}C_{11} \cdot 0.073^{11} \cdot 0.927^{(20-11)} + {}_{20}C_{12} \cdot 0.073^{12} \cdot 0.927^{(20-12)} + {}_{20}C_{13} \cdot 0.073^{13} \cdot 0.927^{(20-13)} + {}_{20}C_{14} \cdot 0.073^{14} \cdot 0.927^{(20-14)} = 0.5923$



27. The Lee family had 6 children. Assuming that the probability of a child being a girl is 0.5, find the probability that the Smith family had at least 4 girls? Binomial, n = 6, p = 0.5, $P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6) = {}_6C_4 \cdot 0.5^4 \cdot 0.5^2 + {}_6C_5 \cdot 0.5^5 \cdot 0.5^1 + {}_6C_6 \cdot 0.5^6 \cdot 0.5^0 = 0.3438$

29. A manufacturing machine has a 6% defect rate. An inspector chooses 4 items at random. Binomial, n = 4, p = 0.06

a) What is the probability that at least one will have a defect? $P(X \geq 1) = 1 - P(X = 0) = 1 - {}_4C_0 \cdot 0.06^0 \cdot 0.94^{(4-0)} = 0.2193$

b) What is the probability that exactly two will have a defect? $P(X = 2) = {}_4C_2 \cdot 0.06^2 \cdot 0.94^{(4-2)} = 0.0191$

c) What is the probability that less than two will have a defect? $P(X < 2) = P(X = 0) + P(X = 1) = {}_4C_0 \cdot 0.06^0 \cdot 0.94^{(4-0)} + {}_4C_1 \cdot 0.06^1 \cdot 0.94^{(4-1)} = 0.9801$

d) What is the probability that more than one will have a defect? $P(X > 1) = 1 - P(X \le 1) = 1 - ({}_4C_0 \cdot 0.06^0 \cdot 0.94^{(4-0)} + {}_4C_1 \cdot 0.06^1 \cdot 0.94^{(4-1)}) = 0.0199$

31. A small regional carrier accepted 20 reservations for a particular flight with 17 seats. 15 reservations went to regular customers who will arrive for the flight. Each of the remaining passengers will arrive for the flight with a 60% chance, independently of each other. Binomial, $n = 5$, $p = 0.6$

a) Find the probability that overbooking occurs. $P(X > 2) = P(X = 3) + P(X = 4) + P(X = 5) = {}_5C_3 \cdot 0.6^3 \cdot 0.4^2 + {}_5C_4 \cdot 0.6^4 \cdot 0.4^1 + {}_5C_5 \cdot 0.6^5 \cdot 0.4^0 = 0.6826$

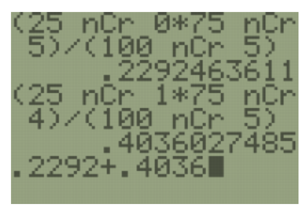
b) Find the probability that the flight has empty seats. $P(X < 2) = {}_5C_0 \cdot 0.6^0 \cdot 0.4^5 + {}_5C_1 \cdot 0.6^1 \cdot 0.4^4 = 0.087$

33. A committee of 5 people is to be formed from 10 students and 7 parents. Find the probability that the committee will consist of exactly 3 students and 2 parents.

Hypergeometric $P(X = x) = \frac{{}_aC_x \cdot {}_bC_{n-x}}{{}_NC_n}$ $N = 17$, $n = 5$, $a = 10$, $b = 7$. $P(X = 3) = \frac{{}_{10}C_3 \cdot {}_7C_2}{{}_{17}C_5} = 0.4072$

35. A bag contains 9 strawberry Starbursts and 21 other flavored Starbursts. 5 Starbursts are chosen randomly without replacement. Find the probability that 3 of the Starbursts drawn are strawberry. Hypergeometric $N = 30$, $n = 5$, $a = 9$, $b = 21$. $P(X = 3) = \frac{{}_9C_3 \cdot {}_{21}C_2}{{}_{30}C_5} = 0.1238$

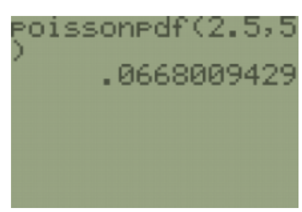
37. A pharmaceutical company receives large shipments of ibuprofen tablets and uses this acceptance sampling plan: randomly select and test 25 tablets, then accept the whole batch if there is at most one that doesn't meet the required specifications. If a particular shipment of 100 ibuprofen tablets actually has 5 tablets that have defects, what is the probability that this whole shipment will be accepted? Hypergeometric $N = 100$, $n = 5$, $a = 25$, $b = 75$. $P(X \le 1) = P(X = 0) + P(X = 1) = \frac{{}_{25}C_0 \cdot {}_{75}C_5}{{}_{100}C_5} + \frac{{}_{25}C_1 \cdot {}_{75}C_4}{{}_{100}C_5} = 0.6328$



39. A writer makes on average one typographical error every page. The writer has landed a 3-page article in an important magazine. If the magazine editor finds any typographical errors, they probably will not ask the writer for any more material. What is the probability that the reporter made no typographical errors for the 3-page article? Poisson $P(X = x) = \frac{e^{-\mu}\mu^x}{x!}$,

$\mu = 1$ typo* $\left(\frac{1\ page}{3\ pages}\right) = 3$ typos, $P(x = 0) = \frac{e^{-3}3^0}{0!} = 0.0498$

41. Suppose a random variable, x, follows a Poisson distribution. Let $\mu = 2.5$ every minute, find the following probabilities.



a) $P(X = 5)$ over a minute. $P(x = 5) = \frac{e^{-2.5}2.5^5}{5!} = 0.0668$

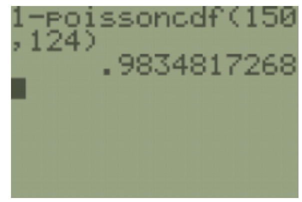
b) P(X < 5) over a minute. P(x < 5) = $\frac{e^{-2.5}2.5^0}{0!} + \frac{e^{-2.5}2.5^1}{1!} + \frac{e^{-2.5}2.5^2}{2!} + \frac{e^{-2.5}2.5^3}{3!} + \frac{e^{-2.5}2.5^4}{4!}$ = 0.8912, TI-84: poissoncdf(2.5,4); Excel: =POISSON.DIST(4,2.5,TRUE)

c) P(X ≤ 5) over a minute. P(x ≤ 5) = $\frac{e^{-2.5}2.5^0}{0!} + \frac{e^{-2.5}2.5^1}{1!} + \frac{e^{-2.5}2.5^2}{2!} + \frac{e^{-2.5}2.5^3}{3!} + \frac{e^{-2.5}2.5^4}{4!} + \frac{e^{-2.5}2.5^5}{5!}$ = 0.9580, TI-84: poissoncdf(2.5,5); Excel: =POISSON.DIST(5,2.5,TRUE)

d) P(X > 5) over a minute. P(X > 5) = 1 – P(X ≤ 5) = 0.0420

e) P(X ≥ 5) over a minute. P(X ≥ 5) = 1 – P(X ≤ 4) = 0.1088

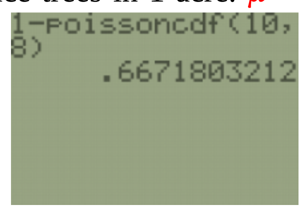
f) P(X = 125) over an hour. $\mu = 2.5\left(\frac{60\ minutes}{1\ minute}\right) = 150$, P(X = 125) = $\frac{e^{-150}150^{125}}{125!}$ = 0.0039



g) P(X ≥ 125) over an hour. P(X ≥ 125) = 1 – P(X ≤ 124) = 1 – poissoncdf(150,124) = 0.9835

43. There are on average 5 old-growth Sitka Spruce trees per 1/8 of an acre in a local forest. Poisson $\mu = 5$ per 1/8 acre

a) Compute the probability that that there are exactly 30 Sitka Spruce trees in 1 acre. $\mu = 5\left(\frac{1\ acre}{0.125\ acre}\right) = 40$, P(X = 30) = $\frac{e^{-40}40^{30}}{30!}$ = 0.0185



b) Compute the probability that that there are more than 8 Sitka Spruce trees in a ¼ acre. $\mu = 5\left(\frac{0.25acre}{0.125\ acre}\right) = 10$, P(X > 8) = 1 – P(X ≤ 8) = 0.6672

45. Suppose a random variable, *x*, follows a Poisson distribution. Let μ = 3 every day; compute P(X ≤ 12) over a week. $\mu = \left(3\frac{7\ days}{1\ day}\right) = 21$, P(X ≤ 12) = 0.0245

Chapter 6 Exercises

1. The waiting time for a bus is uniformly distributed between 0 and 15 minutes. What is the probability that a person has to wait at most 8 minutes for a bus? $P(X \leq 8) = \left(\frac{1}{15-0}\right) \cdot (8-0) = \frac{8}{15} = 0.5333$

3. The time it takes me to wash the dishes is uniformly distributed between 7 minutes and 16 minutes. What is the probability that washing dishes tonight will take me between 9 and 14 minutes? $P(9 \leq X \leq 14) = \left(\frac{1}{16-7}\right) \cdot (14-9) = \frac{5}{9} = 0.5556$

5. The lengths of an instructor's classes have a continuous uniform distribution between 60 and 90 minutes. If one such class is randomly selected, find the probability that the class length is less than 70.7 minutes. $P(X < 70.7) = \left(\frac{1}{90-60}\right) \cdot (70.7-60) = \frac{10.7}{30} = 0.3567$

7. Suppose the commuter trains of a public transit system have a waiting time during peak rush hour periods of fifteen minutes. Assume the waiting times are uniformly distributed. Find the probability of waiting between 7 and 8 minutes. $P(7 \leq X \leq 8) = \left(\frac{1}{15-0}\right) \cdot (8-7) = 0.0667$

9. The amount of gasoline sold daily at a service station is uniformly distributed with a minimum of 2,000 gallons and a maximum of 5,000 gallons.

   a) Compute the probability that the service station will sell at least 4,000 gallons? $P(X \geq 4000) = \left(\frac{1}{5000-2000}\right) \cdot (5000-4000) = \frac{1000}{3000} = 0.3333$
   b) Compute the probability that daily sales will fall between 2,500 and 3,000 gallons? $P(2500 \leq X \leq 3000) = \left(\frac{1}{5000-2000}\right) \cdot (3000-2500) = \frac{500}{3000} = 0.1667$
   c) What is the probability that the station will sell exactly 2,500 gallons? 0

11. Suppose that the distance, in miles, that people are willing to commute to work is exponentially distributed with mean 24 miles. What is the probability that people are willing to commute at most 12 miles to work? $P(X \leq 12) = 1 - e^{-12/24} = 0.3935$

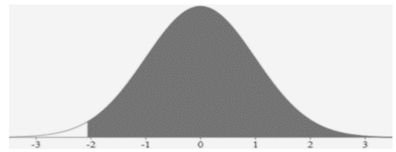
13. The lifetime of an LED lightbulb is exponentially distributed with an average lifetime of 5,000 hours. Find the following.
   a) The probability that the LED lightbulb will last more than 3,500 hours. $P(X > 3500) = e^{-3500/5000} = 0.4966$
   b) The probability that the LED lightbulb will last less than 4,000 hours. $P(X < 4000) = 1 - e^{-4000/5000} = 0.5507$
   c) The probability that the LED lightbulb will last between 3,500 and 4,500 hours. $P(3500 \leq X \leq 4500) = e^{(-3500/5000)} - e^{(-4500/5000)} = 0.09$

15. The average time it takes a salesperson to finish a sale on the phone is 5 minutes and is exponentially distributed.

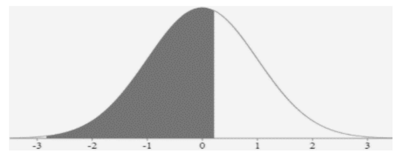
a) Compute the probability that less than 10 minutes pass before a sale is completed. P(X < 10) = $1 - e^{-10/5}$ = 0.8647
b) Compute the probability that more than 15 minutes pass before a sale is completed. . P(X > 15) = $e^{-15/5}$ = 0.0498
c) Compute the probability that between 10 and 15 minutes pass before a sale is completed. $P(10 \leq X \leq 15) = e^{(-10/5)} - e^{(-15/5)} = 0.0855$

17. For a standard normal distribution, find the following probabilities.

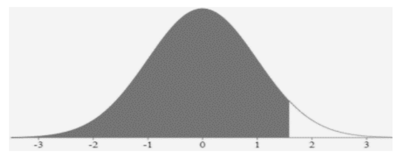
a) P(Z > -2.06)
Excel: =1-NORM.S.DIST(-2.06,TRUE) = 0.9803
TI-84: normalcdf(-2.06,1E99,0,1) = 0.9803



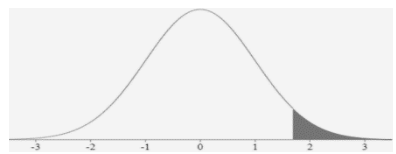
b) P(-2.83 < Z < 0.21)
Excel: =NORM.S.DIST(0.21,TRUE)-NORM.S.DIST(-2.83,TRUE) = 0.5809
TI-84: normalcdf(-2.83,0.21,0,1) = 0.5809



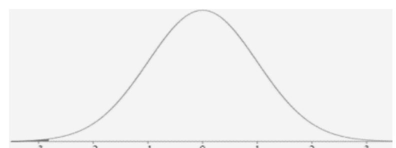
c) P(Z < 1.58)
Excel: =NORM.S.DIST(1.58,TRUE) = 0.9429
TI-84: normalcdf(-1E99,1.58,0,1) = 0.9429



d) P(Z ≥ 1.69)
Excel: =1-NORM.S.DIST(1.69,TRUE) = 0.0455
TI-84: normalcdf(1.69,1E99,0,1) = 0.0455

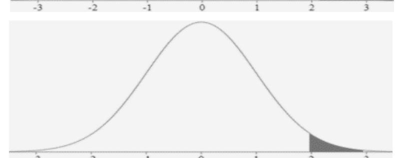


e) P(Z < -2.82)
Excel: =NORM.S.DIST(-2.82,TRUE) = 0.0024
TI-84: normalcdf(-1E99,-2.82,0,1) = 0.0024

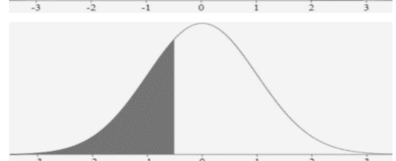


f) P(Z > 2.14)
Excel: =1-NORM.S.DIST(2.14,TRUE) = 0.0162
TI-84: normalcdf(2.14,1E99,0,1) = 0.0162

g) P(1.97 ≤ Z ≤ 2.93)
Excel: =NORM.S.DIST(2.93,TRUE)-NORM.S.DIST(1.97,TRUE) = 0.0227
TI-84: normalcdf(1.97,2.93,0,1) = 0.0227

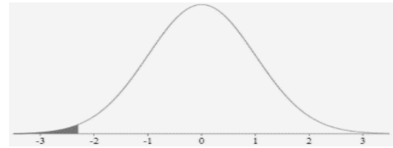


h) P(Z ≤ -0.51)
Excel: =NORM.S.DIST(-0.51,TRUE) = 0.305
TI-84: normalcdf(-1E99,-0.51,0,1) = 0.305

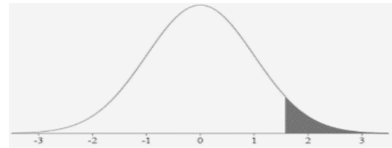


19. Compute the following probabilities where $Z \sim N(0,1)$.

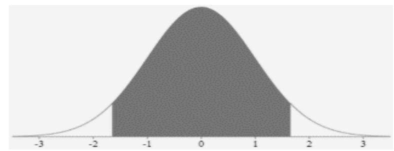
a) $P(Z \leq -2.03)$
Excel: =NORM.S.DIST(-2.03,TRUE) = 0.0107
TI-84: normalcdf(-1E99,-2.03,0,1) = 0.0107



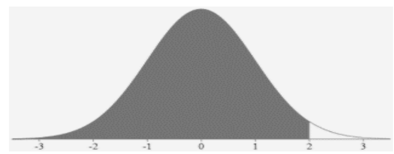
b) $P(Z > 1.58)$
Excel: =1-NORM.S.DIST(1.58,TRUE) = 0.0571
TI-84: normalcdf(1.58,1E99,0,1) = 0.0571



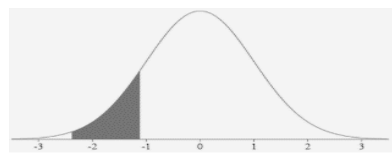
c) $P(-1.645 \leq Z \leq 1.645)$
Excel: =NORM.S.DIST(1.645,TRUE)-NORM.S.DIST(-1.645,TRUE) = 0.90
TI-84: normalcdf(-1.645,1.645,0,1) = 0.90



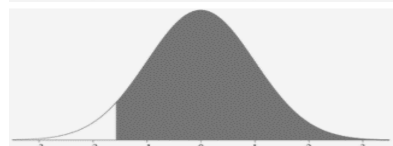
d) $P(Z < 2)$
Excel: =NORM.S.DIST(2,TRUE) = 0.9772
TI-84: normalcdf(-1E99,2,0,1) = 0.9772

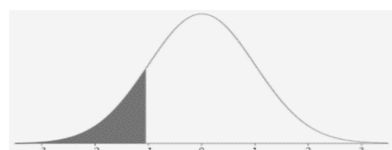


e) $P(-2.38 < Z < -1.12)$
Excel: =NORM.S.DIST(-1.12,TRUE)-NORM.S.DIST(-2.38,TRUE) = 0.1227
TI-84: normalcdf(-2.38,-1.12,0,1) = 0.1227



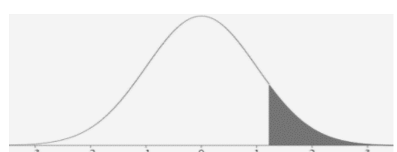
f) $P(Z \geq -1.75)$
Excel: =1-NORM.S.DIST(-1.57,TRUE) = 0.9418
TI-84: normalcdf(-1.57,1E99,0,1) = 0.9418





21. Compute the area under the standard normal distribution to the left of z = -0.69.
Excel: =NORM.S.DIST(-0.69,TRUE) = 0.2451
TI-84: normalcdf(-1E99,-0.69,0,1) = 0.2451

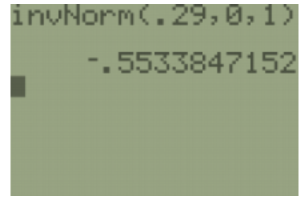
23. Compute the area under the standard normal distribution to the right of z = 1.22.
Excel: =1-NORM.S.DIST(1.22,TRUE) = 0.1112
TI-84: normalcdf(1.22,1E99,0,1) = 0.1112



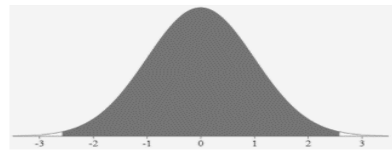
25. Compute the area under the standard normal distribution between z = -2.97 and z = -2.14.
Excel: =NORM.S.DIST(-2.14,TRUE)-NORM.S.DIST(-2.97,TRUE) = 0.0147
TI-84: normalcdf(-2.97,-2.14,0,1) = 0.0147

27. Compute the area under the standard normal distribution to the left of z = 0.85.
    Excel: =NORM.S.DIST(0.85,TRUE) = 0.8023
    TI-84: normalcdf(-1E99,0.85,0,1) = 0.8023
    $P(Z \leq 0.85) = 0.8023$

29. For the standard normal distribution, find the z score that gives the 29th percentile. -0.5534

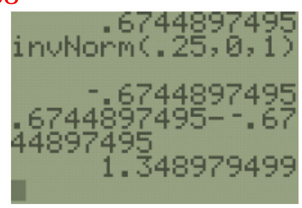


31. Compute the two z-scores that give the middle 99% of the standard normal distribution.
    Area between two unknown z-scores is 0.99, that leaves 1 – 0.99 = 0.01 area split between both tails. Half of 0.01 is 0.005. Use left tail areas 0.005 and 1 – 0.005 = 0.995.

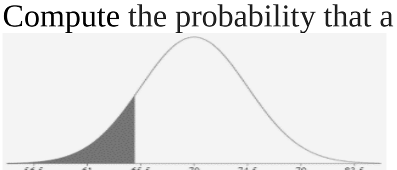


    Excel =NORM.S.INV(0.005) = -2.5758 and =NORM.S.INV(0.995) = 2.5758
    TI-84: invNorm(0.005,0,1) = -2.5758 and invNorm(0.995,0,1) = 2.5758

33. Find the IQR for the standard normal distribution.
    IQR = $Q_3 - Q_1$ = 0.67449 – (-0.67449) = 1.349

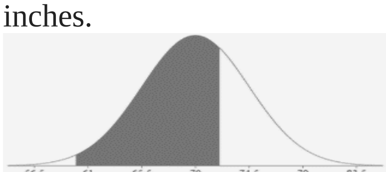


35. Arm span is the physical measurement of the length of an individual's arms from fingertip to fingertip. A man's arm span is approximately normally distributed with mean of 70 inches with a standard deviation of 4.5 inches.
    a) Compute the probability that a randomly selected man has an arm span below 65 inches.



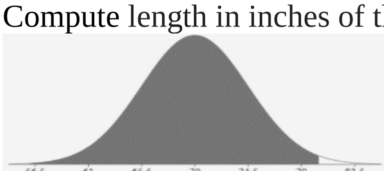
    Excel: =NORM.DIST(65,70,4.5,TRUE) = 0.1333
    TI-84: normalcdf(-1E99,65,70,4.5) = 0.1333

    b) Compute the probability that a randomly selected man has an arm span between 60 and 72 inches.



    Excel: =NORM.DIST(72,70,4.5,TRUE)-NORM.S.DIST(60,70,4.5,TRUE) = 0.6585
    TI-84: normalcdf(60,72,70,4.5) = 0.6585

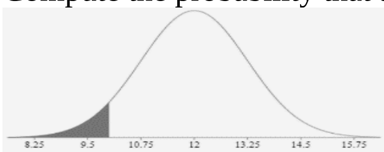
    c) Compute length in inches of the 99th percentile for a man's arm span.



    Excel =NORM.INV(0.99,70,4.5) = 80.4686
    TI-84: invNorm(0.99,70,4.5) = 80.4686

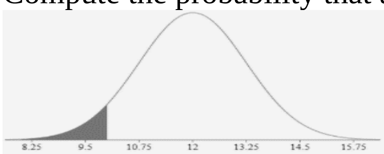
37. A dishwasher has a mean life of 12 years with an estimated standard deviation of 1.25 years ("Appliance life expectancy," 2013). Assume the life of a dishwasher is normally distributed.

   a) Compute the probability that a dishwasher will last less than 10 years.



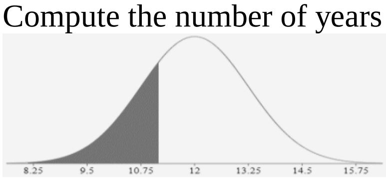
   Excel: =NORM.DIST(10,12,1.25,TRUE) = 0.0548
   TI-84: normalcdf(-1E99,10,12,1.25) = 0.0548

   b) Compute the probability that a dishwasher will last between 8 and 10 years.



   Excel: =NORM.DIST(10,12,1.25,TRUE)-NORM.S.DIST(8,12,1.25,TRUE) = 0.0541
   TI-84: normalcdf(8,10,12,1.25) = 0.0541

   c) Compute the number of years that the bottom 25% of dishwashers would last.



   Excel =NORM.INV(0.25,12,1.25) = 11.1569
   TI-84: invNorm(0.25,12,1.25) = 11.1569 years

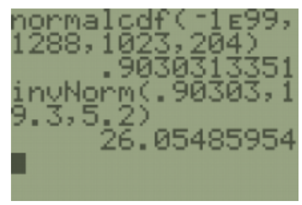
39. Heights of 10-year-old children, regardless of sex, closely follow a normal distribution with mean 55.7 inches and standard deviation 6.8 inches.

   a) Compute the probability that a randomly chosen 10-year-old child is less than 50.4 inches.
   Excel: =NORM.DIST(50.4,55.7,6.8,TRUE) = 0.2179
   TI-84: normalcdf(-1E99,50.4,55.7,6.8) = 0.2179
   b) Compute the probability that a randomly chosen 10-year-old child is more than 59.2 inches.
   Excel: =1-NORM.DIST(59.2,55.7,6.8,TRUE) = 0.3034
   TI-84: normalcdf(59.2,1E99,55.7,6.8) = 0.3034
   c) What proportion of 10-year-old children are between 50.4 and 61.5 inches tall? 0.5853
   Excel: =NORM.DIST(61.5,55.7,6.8,TRUE)-NORM.S.DIST(50.4,55.7,6.8,TRUE) = 0.0541
   TI-84: normalcdf(50.4,61.5,55.7,6.8) = 0.0541
   d) Compute the 85^{th} percentile for 10-year-old children.
   Excel =NORM.INV(0.85,55.7,6.8) = 62.7477 inches
   TI-84: invNorm(0.85,55.7,6.8) = 62.7477 inches

41. The mean daily milk production of a herd of cows is assumed to be normally distributed with a mean of 33 liters, and standard deviation of 10.3 liters. Compute the probability that daily production is more than 40.9 liters?
Excel: =1-NORM.DIST(40.9,33,10.3,TRUE) = 0.2215
TI-84: normalcdf(40.9,1E99,33,10.3) = 0.2215

43. A study was conducted on students from a particular high school over the last 8 years. The following information was found regarding standardized tests used for college admittance. Scores on the SAT test are normally distributed with a mean of 1023 and a standard deviation of 204. Scores on the ACT test are normally distributed with a mean of 19.3 and a standard deviation of 5.2. It is assumed that the two tests measure the same aptitude, but use different scales.

a) Compute the SAT score that is the 50-percentile.
Excel =NORM.INV(0.5,1023,204) = 1023
TI-84: invNorm(0.5,1023,204) = 1023
b) Compute the ACT score that is the 50-percentile.
Excel =NORM.INV(0.5,19.3,5.2) = 19.3
TI-84: invNorm(0.5,19.3,5.2) = 19.3
c) If a student gets an SAT score of 1288, find their equivalent ACT score. Go out at least 5 decimal places between steps.
$P(X \leq 1288)$ = normcdf(1E99,1288,1023,204) = 0.90303
invNorm(0.90303,19.3,5.2) = 26.1



45. The MAX light rail in Portland, OR has a waiting time that is uniformly distributed with a mean waiting time of 5 minutes with a standard deviation of 2.9 minutes. A random sample of 40 wait times was selected. What is the probability the sample **mean** wait time is under 4 minutes?

Use the Central Limit Theorem to find the probability of a mean $P(\bar{x} < 4)$. Even though the distribution of the population is uniform, the sampling distribution will be normally distributed with a mean of 5 and a standard deviation of $\frac{2.9}{\sqrt{40}}$ since the sample size is over 30.
Excel: =NORM.DIST(4,5,2.9/SQRT(40),TRUE) = 0.0146
TI-84: normalcdf(-1E99,4,5,2.9/$\sqrt{40}$) = 0.0146

47. A certain brand of electric bulbs has an average life of 300 hours with a standard deviation of 45. A random sample of 100 bulbs is tested. What is the probability that the sample **mean** will be less than 295?

Use the Central Limit Theorem to find the probability of a mean $P(\bar{x} < 295)$. Even though the distribution of the population is unknown, the sample size is over 30. The sampling distribution will be normally distributed with a mean of 300 and a standard deviation of $\frac{45}{\sqrt{100}}$.
Excel: =NORM.DIST(295,300,45/SQRT(100),TRUE) = 0.1333
TI-84: normalcdf(-1E99,295,300,45/$\sqrt{100}$) = 0.1333

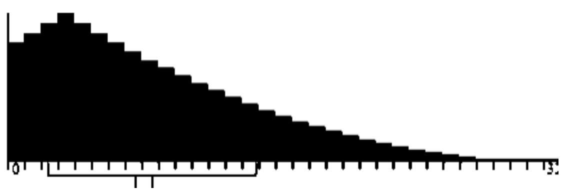
49. If the Central Limit Theorem is applicable, this means that the sampling distribution of a ____ _________ population can be treated as normal since the ___________ is __________ .

a) symmetrical; variance; large
b) positively skewed; sample size; small
c) negatively skewed; standard deviation; large
d) non-normal; mean; large
e) negatively skewed; sample size; large

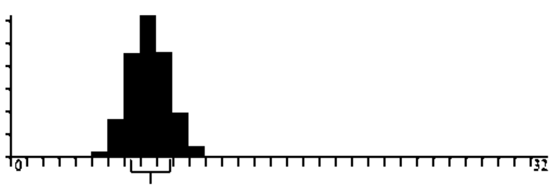
Answer e). If the Central Limit Theorem is applicable, this means that the sampling distribution of a negatively skewed population can be treated as normal since the sample size is large.

51. Match the following 3 graphs with the distribution of the population, the distribution of the sample, and the sampling distribution.

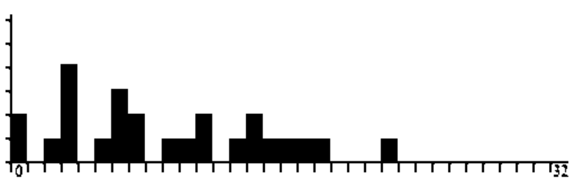
a) Distribution of the Population
b) Distribution of the Sample
c) Sampling Distribution



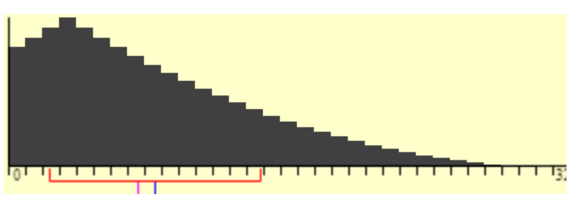
i.



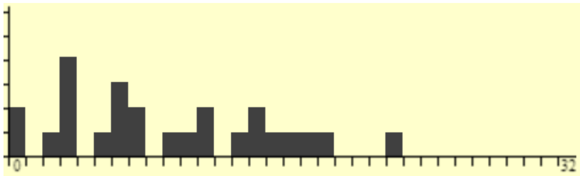
ii.



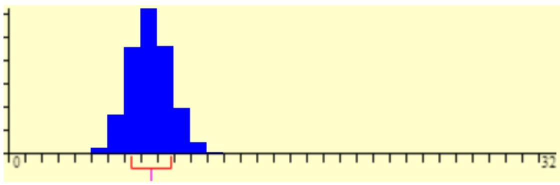
iii.



Distribution of the Population



Distribution of the Sample



Sampling Distribution

Chapter 7 Exercises

1. Which confidence level would give the narrowest margin of error? Answer a) 80%
   a) 80%
   b) 90%
   c) 95%
   d) 99%

2. Suppose you compute a confidence interval with a sample size of 25. What will happen to the width of the confidence interval if the sample size increases to 50, assuming everything else stays the same? Choose the correct answer below. Answer a)
   a) Gets smaller
   b) Stays the same
   c) Gets larger

3. For a confidence level of 90% with a sample size of 35, find the critical z values. Use technology, invNorm(0.05,0,1), z = ±1.6449

5. A researcher would like to estimate the proportion of all children that have been diagnosed with autism spectrum disorder (ASD) in their county. They are using 95% confidence level and the CDC 2018 national estimate that 1 in 68 ≈ 0.0147 children are diagnosed with ASD. What sample size should the researcher use to get a margin of error to be within 2%? Round up to the nearest integer. invNorm(0.025,0,1) = –1.959963986
   $n = p^* \cdot q^* \left(\frac{z_{\alpha/2}}{E}\right)^2 = \left(\frac{1}{68}\right)\left(\frac{67}{68}\right)\left(\frac{-1.959963986}{0.02}\right)^2 = 139.15$ Always round up so use n = 140

7. A pilot study found that 72% of adult Americans would like an Internet connection in their car.

   a) Use the given preliminary estimate to determine the sample size required to estimate the proportion of adult Americans who would like an Internet connection in their car to within 0.02 with 95% confidence. $n = p^* \cdot q^* \left(\frac{z_{\alpha/2}}{E}\right)^2 = (0.72)(0.28)\left(\frac{-1.959963986}{0.02}\right)^2 =$ 1936.095, always round up, so use n = 1937.
   b) Use the given preliminary estimate to determine the sample size required to estimate the proportion of adult Americans who would like an Internet connection in their car to within 0.02 with 99% confidence. $n = p^* \cdot q^* \left(\frac{z_{\alpha/2}}{E}\right)^2 = (0.72)(0.28)\left(\frac{-2.575829303}{0.02}\right)^2 =$ 3343.98, always round up, so use n = 3344.
   c) If the information in the pilot study was not given, determine the sample size required to estimate the proportion of adult Americans who would like an Internet connection in their car to within 0.02 with 99% confidence. $n = p^* \cdot q^* \left(\frac{z_{\alpha/2}}{E}\right)^2 = (0.5)(0.5)\left(\frac{-2.575829303}{0.02}\right)^2$ = 4146.81, always round up, so use n = 4147.

9. In a random sample of 200 people, 135 said that they watched educational TV. Find and interpret the 95% confidence interval of the true proportion of people who watched educational TV.

$\hat{p} = \frac{x}{n} = \frac{135}{200} = 0.675 \qquad \hat{q} = 1 - \hat{p} = 1 - 0.675 = 0.325$

invNorm(0.025,0,1) = –1.959963986

$\hat{p} \pm z_{\alpha/2}\sqrt{\left(\frac{\hat{p}\hat{q}}{n}\right)} \qquad 0.675 \pm -1.959963986\sqrt{\left(\frac{0.675 \cdot 0.325}{200}\right)}$ $\qquad 0.6101 < p < 0.7399$

11. A teacher wanted to estimate the proportion of students who take notes in her class. She used data from a random sample size of 82 and found that 50 of them took notes. The 99% confidence interval for the proportion of student that take notes is _____< p < ________.

$\hat{p} = \frac{x}{n} = \frac{50}{82} = 0.6098 \qquad \hat{q} = 1 - \hat{p} = 1 - 0.6098 = 0.3902$

invNorm(0.005,0,1) = –2.5758

$\hat{p} \pm z_{\alpha/2}\sqrt{\left(\frac{\hat{p}\hat{q}}{n}\right)} \qquad 0.6098 \pm -2.5758\sqrt{\left(\frac{0.6098 \cdot 0.3902}{82}\right)}$ $\qquad 0.4710 < p < 0.749$

13. A survey asked people if they were aware that maintaining a healthy weight could reduce the risk of stroke. A 95% confidence interval was found using the survey results to be (0.54, 0.62). Which of the following is the correct interpretation of this interval? Answer a)
    a) We are 95% confident that the interval 0.54 < p < 0.62 contains the population proportion of people who are aware that maintaining a healthy weight could reduce the risk of stroke.
    b) There is a 95% chance that the sample proportion of people who are aware that maintaining a healthy weight could reduce the risk of stroke is between 0.54 < p < 0.62.
    c) There is a 95% chance of having a stroke if you do not maintain a healthy weight.
    d) There is a 95% chance that the proportion of people who will have a stroke is between 54% and 62%.

15. A laboratory in Florida is interested in finding the mean chloride level for a healthy resident in the state. A random sample of 25 healthy residents has a mean chloride level of 80 mEq/L. If it is known that the chloride levels in healthy individuals residing in Florida is normally distributed with a population standard deviation of 27 mEq/L, find and interpret the 95% confidence interval for the true mean chloride level of all healthy Florida residents.

invNorm(0.025,0,1) = –1.959963986 $\qquad \bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \qquad 80 \pm -1.959963986\left(\frac{27}{\sqrt{25}}\right)$

69.416 < μ < 90.584

We can be 95% confident that the population mean chloride level in Florida is between 69.416 and 90.584 mEq/L.

17. The age when smokers first start from previous studies is normally distributed with a mean of 13 years old with a population standard deviation of 2.1 years old. A survey of smokers of this generation was done to estimate if the mean age has changed. The sample of 33 smokers found that their mean starting age was 13.7 years old. Find the 99% confidence interval of the mean.

Use a z-interval since the population standard deviation was given.

invNorm(0.005,0,1) = –2.575829303 $\qquad \bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \qquad 13.7 \pm -2.575829303\left(\frac{2.1}{\sqrt{33}}\right)$

12.7584 < μ < 14.6416

19. The undergraduate grade point average (GPA) for students admitted to the top graduate business schools was 3.53. Assume this estimate was based on a sample of 8 students admitted

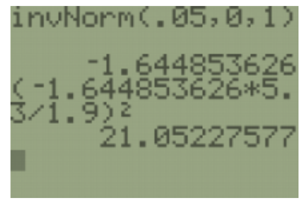
to the top schools. Assume that the population is normally distributed with a standard deviation of 0.18. Find and interpret the 99% confidence interval estimate of the mean undergraduate GPA for all students admitted to the top graduate business schools.

invNorm(0.005,0,1) = –2.575829303 $\qquad \bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

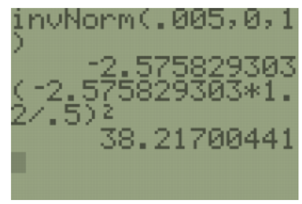
$3.53 \pm -2.575829303\left(\frac{0.18}{\sqrt{8}}\right)$ $\qquad 3.3611 < \mu < 3.6939$

We can be 99% confident that the population mean GPA for all graduate business students admitted into the top schools is between 3.3611 and 3.6939.

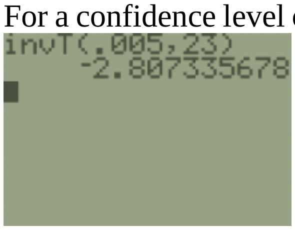
21. You want to obtain a sample to estimate a population mean age of the incoming fall term transfer students. Based on previous evidence, you believe the population standard deviation is approximately 5.3. You would like to be 90% confident that your estimate is within 1.9 of the true population mean. How large of a sample size is required? $n = \left(\frac{z_{\alpha/2}\cdot\sigma}{E}\right)^2 = \left(\frac{-1.644853626\cdot 5.3}{1.9}\right)^2 = 21.05$ Always round up so use $n = 22$



23. An engineer wishes to determine the width of a particular electronic component. If she knows that the standard deviation is 1.2 mm, how many of these components should she consider to be 99% sure of knowing the mean will be within 0.5 mm? $n = \left(\frac{z_{\alpha/2}\cdot\sigma}{E}\right)^2 = \left(\frac{-2.575829303\cdot 1.2}{0.5}\right)^2 = 38.217$, round up to $n = 39$.



25. For a confidence level of 99% with a sample size of 24, find the critical t values. t = ±2.807336



27. The amount of money in the money market accounts of 26 customers is found to be approximately normally distributed with a mean of $18,240 and a sample standard deviation of $1,100. Find and interpret the 95% confidence interval for the mean amount of money in the money market accounts at this bank.

invT(0.025,25) = –2.059539 $\qquad \bar{x} \pm t_{\alpha/2,n-1}\left(\frac{s}{\sqrt{n}}\right) \qquad 18240 \pm -2.059539\left(\frac{1100}{\sqrt{26}}\right)$

$17795.7087 < \mu < 18684.2913$

We can be 95% confident that the population mean amount of money in all money market accounts is between $17,795.71 and $18,684.29.

29. A random sample of stock prices per share (in dollars) is shown. Find and interpret the 90% confidence interval for the mean stock price. Assume the population of stock prices is normally distributed.

26.60 75.37 3.81 28.37 40.25 13.88 53.80 28.25 10.87 12.25

invT(0.05,9) = –1.833113 $\bar{x} \pm t_{\alpha/2,n-1}\left(\frac{s}{\sqrt{n}}\right)$ $29.345 \pm -1.833113\left(\frac{22.034615}{\sqrt{10}}\right)$

16.5720 < μ < 42.1180 We can be 90% confident that the population mean stock price is between \$16.57 and \$42.12.

31. A tire manufacturer wants to estimate the average number of miles that may be driven in a tire of a certain type before the tire wears out. Assume the population is normally distributed. A random sample of tires is chosen and are driven until they wear out and the number of thousands of miles is recorded, find and interpret the 99% confidence interval for the mean using the sample data: 32, 33, 28, 37, 29, 30, 22, 35, 23, 28, 30, 36.

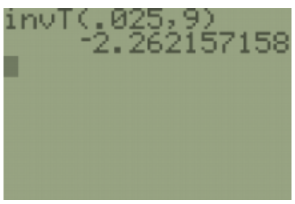
invT(0.005,11) = –3.105807 $\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$ $30.25 \pm -3.105807\left(\frac{4.71217}{\sqrt{12}}\right)$

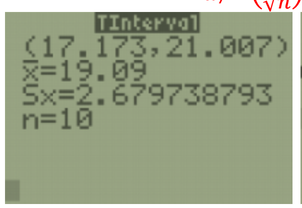
26.0252 < μ < 34.47478 We can be 99% confident that the population mean lifetime for this type of tire is between 26,025 and 34,475 miles.

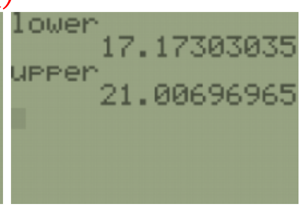
33. A sample of the length in inches for newborns is given below. Assume that lengths are normally distributed. Find the 95% confidence interval of the mean length.

Length	20.8	16.9	21.9	18	15	20.8	15.2	22.4	19.4	20.5

invT(0.025,9) = –2.262157 $\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$ $19.09 \pm -2.262157\left(\frac{2.679739}{\sqrt{10}}\right)$







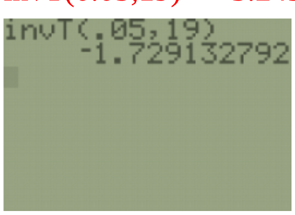
17.1731 < μ < 21.0069

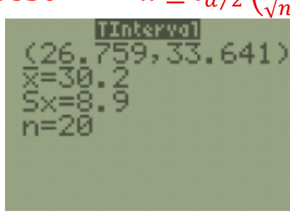
35. Which of the following would result in the widest confidence interval? Answer: d)

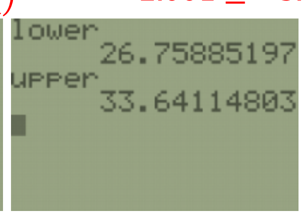
a) A sample size of 100 with 99% confidence.
b) A sample size of 100 with 95% confidence.
c) A sample size of 30 with 95% confidence.
d) A sample size of 30 with 99% confidence.

37. The total of individual weights of garbage discarded by 20 households in one week is normally distributed with a mean of 30.2 lbs with a sample standard deviation of 8.9 lbs. Find the 90% confidence interval of the mean.

invT(0.05,19) = –3.249836 $\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$ $2.001 \pm -3.249836\left(\frac{0.227032}{\sqrt{10}}\right)$







26.7589 < μ < 33.6411

39. A researcher finds a 95% confidence interval for the average commute time in minutes using public transit is (15.75, 28.25). Which of the following is the correct interpretation of this interval? Answer d)
    a) We are 95% confident that all commute time in minutes for the population using public transit is between 15.75 and 28.25 minutes.
    b) There is a 95% chance commute time in minutes using public transit is between 15.75 and 28.25 minutes.
    c) We are 95% confident that the interval $15.75 < \mu < 28.25$ contains the sample mean commute time in minutes using public transportation.
    d) We are 95% confident that the interval $15.75 < \mu < 28.25$ contains the population mean commute time in minutes using public transportation.

Chapter 8 Exercises

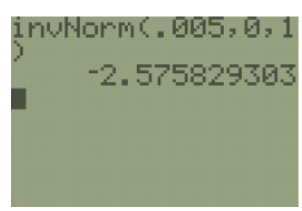
1. The plant-breeding department at a major university developed a new hybrid boysenberry plant called Stumptown Berry. Based on research data, the claim is made that from the time shoots are planted 90 days on average are required to obtain the first berry. A corporation that is interested in marketing the product tests 60 shoots by planting them and recording the number of days before each plant produces its first berry. The sample mean is 92.3 days. The corporation wants to know if the mean number of days is different from the 90 days claimed. Which one is the correct set of hypotheses?
   a) H_0: p = 90% H_1: p ≠ 90%
   b) H_0: μ = 90 H_1: μ ≠ 90
   c) H_0: p = 92.3% H_1: p ≠ 92.3%
   d) H_0: μ = 92.3 H_1: μ ≠ 92.3
   e) H_0: μ ≠ 90 H_1: μ = 90

   Answer: b) H_0: μ = 90 H_1: μ ≠ 90
   Note that your null hypotheses should always contain the signs $=, \leq, or \geq$ and your alternative hypotheses should always contain the signs $\neq, <,\ or >$. Since this problem is testing for an "average," we should use the variable μ in our hypotheses. The problem is specifying that we should test for a "difference" in the number of days, so we will choose $\neq$ for our alternative hypothesis.

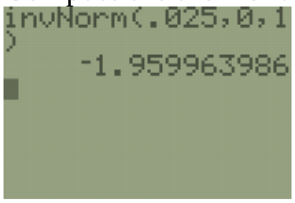
3. According to the February 2008 Federal Trade Commission report on consumer fraud and identity theft, 23% of all complaints in 2007 were for identity theft. In that year, Alaska had 321 complaints of identity theft out of 1,432 consumer complaints. Does this data provide enough evidence to show that Alaska had a lower proportion of identity theft than 23%? Which one is the correct set of hypotheses?
   Federal Trade Commission, (2008). *Consumer fraud and identity theft complaint data: January-December 2007*. Retrieved from website: http://www.ftc.gov/opa/2008/02/fraud.pdf.
   a) H_0: p = 23% H_1: p < 23%
   b) H_0: μ = 23 H_1: μ < 23
   c) H_0: p < 23% H_1: p ≥ 23%
   d) H_0: p = 0.224 H_1: p < 0.224
   e) H_0: μ < 0.224 H_1: μ ≥ 0.224

   Answer: a) H_0: p = 23% H_1: p < 23%
   Note that your null hypotheses should always contain the signs $=, \leq, or\ \geq$ and your alternative hypotheses should always contain the signs $\neq, <,\ or >$. The problem pertains to proportions, so we will use the variable *p* in the hypotheses. It also specifies that we should test for a "lower" proportion, so we will select the sign of < for the alternative hypothesis.

5. Compute the z critical value for a two-tailed test when α = 0.01.
   z = invNorm(0.005, 0, 1) = ±2.5758



7. Compute the the z critical value for a two-tailed test when $\alpha = 0.05$.



$z = \text{invNorm}(0.025, 0, 1) = \pm 1.96$

9. Match the phrase with the correct symbol.

a.	Sample Size	i.	α
b.	Population Mean	ii.	n
c.	Sample Variance	iii.	σ^2
d.	Sample Mean	iv.	s^2
e.	Population Standard Deviation	v.	s
f.	P(Type I Error)	vi.	$\bar{x}$
g.	Sample Standard Deviation	vii.	σ
h.	Population Variance	viii.	μ

n = Sample Size
μ = Population Mean
s^2 = Sample Variance
$\bar{x}$ = Sample Mean
σ = Population Standard Deviation
α = P(Type I Error)
s = Sample Standard Deviation
σ^2 = Population Variance

11. Match the symbol with the correct phrase.

$100(1-\alpha)\%$	Parameter
$1-\beta$	P(Type II Error)
β	Power
μ	Significance Level
α	Confidence Level

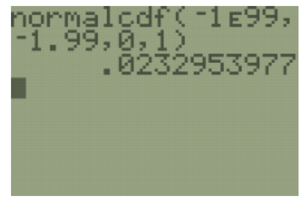
$100(1-\alpha)\%$	Confidence Level
$1-\beta$	Power
β	P(Type II Error)
μ	Parameter
α	Significance Level

13. The Food & Drug Administration (FDA) regulates that fresh albacore tuna fish contains at most 0.82 ppm of mercury. A scientist at the FDA believes the mean amount of mercury in tuna fish for a new company exceeds the ppm of mercury. The hypotheses are H_0: $\mu = 0.82$ H_1: $\mu > 0.82$. Which answer is the correct type II error in the context of this problem?
   a) The fish is rejected by the FDA when in fact it had less than 0.82 ppm of mercury.

b) The fish is accepted by the FDA when in fact it had less than 0.82 ppm of mercury.
c) The fish is rejected by the FDA when in fact it had more than 0.82 ppm of mercury.
d) The fish is accepted by the FDA when in fact it had more than 0.82 ppm of mercury.

Answer: d)
A Type 2 Error occurs when the null hypothesis is not rejected, although it should have been. In this case, that means that it was determined the mercury level was less than or equal to 0.82 ppm, but in fact it was actually greater than 0.82 ppm.

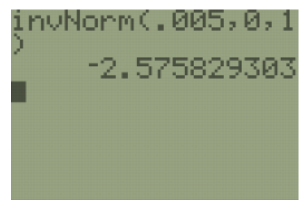
15. A left-tailed z-test found a test statistic of z = -1.99. At a 5% level of significance, what would the correct decision be?



a) Do not reject H_0
b) Reject H_0
c) Accept H_0
d) Reject H_1
e) Do not reject H_1

Answer: b)
Find the p-value in your calculator using: normalcdf(-1000000, -1.99, 0, 1).
Since $p < \alpha$, reject H_0.

17. A two-tailed z-test found a test statistic of z = -2.19. At a 1% level of significance, which would the correct decision?



a) Do not reject H_0
b) Reject H_0
c) Accept H_0
d) Reject H_1
e) Do not reject H_1

Answer: a)
The critical values for $\alpha = 0.01$ are $z = \pm 2.5757$. Do not reject H_0 since the test statistic is not in the critical rejecting region.

19. A hypothesis test was conducted during a clinical trial to see if a new COVID-19 vaccination reduces the risk of contracting the virus. What is the Type I and II errors in terms of approving the vaccine for use?

The implication of a Type I error from the clinical trial is that the vaccination will be approved when it indeed does not reduce the risk of contracting the virus.

The implication of a Type II error from the clinical trial is that the vaccination will not be approved when it indeed does reduce the risk of contracting the virus.

21. A commonly cited standard for one-way length (duration) of school bus rides for elementary school children is 30 minutes. A local government office in a rural area conducts a study to determine if elementary schoolers in their district have a longer average one-way commute time. If they determine that the average commute time of students in their district is

significantly higher than the commonly cited standard they will invest in increasing the number of school buses to help shorten commute time. What would a Type II error mean in this context?

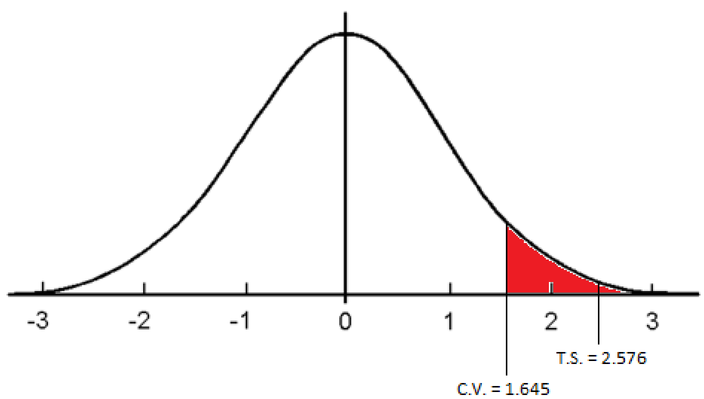
The local government decides that the data do not provide convincing evidence of an average commute time higher than 30 minutes, when the true average commute time is in fact higher than 30 minutes.

23. The Food & Drug Administration (FDA) regulates that fresh albacore tuna fish contains at most 0.82 ppm of mercury. A scientist at the FDA believes the mean amount of mercury in tuna fish for a new company exceeds the ppm of mercury. A test statistic was found to be 2.576 and a critical value was found to be 1.645, what is the correct decision and summary?
   a) Reject H_0, there is enough evidence to support the claim that the amount of mercury in the new company's tuna fish exceeds the FDA limit of 0.82 ppm.
   b) Accept H_0, there is not enough evidence to reject the claim that the amount of mercury in the new company's tuna fish exceeds the FDA limit of 0.82 ppm.
   c) Reject H_1, there is not enough evidence to reject the claim that the amount of mercury in the new company's tuna fish exceeds the FDA limit of 0.82 ppm.
   d) Reject H_0, there is not enough evidence to support the claim that the amount of mercury in the new company's tuna fish exceeds the FDA limit of 0.82 ppm.
   e) Do not reject H_0, there is not enough evidence to support the claim that the amount of mercury in the new company's tuna fish exceeds the FDA limit of 0.82 ppm.

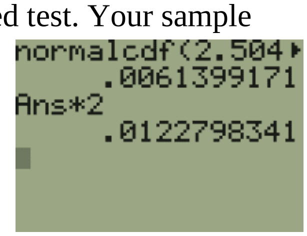
Answer: a)

Plot the critical value on a graph and shade to the right of it since this is a right tailed test (see below). Plot the test statistic.

Since the test statistic lands in the shaded region, we reject the null hypothesis, and thus, we are supporting the claim in the alternative hypothesis – that the mercury level exceeds 0.82 ppm.

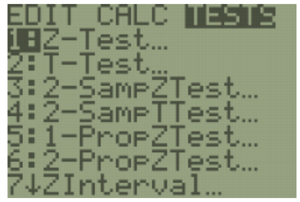


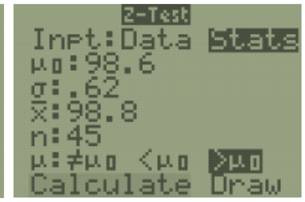
25. You are conducting a study to see if the accuracy rate for fingerprint identification is significantly different from 0.34. Thus, you are performing a two-tailed test. Your sample data produce the test statistic z = 2.504. Use your calculator to find the p-value and state the correct decision and summary.

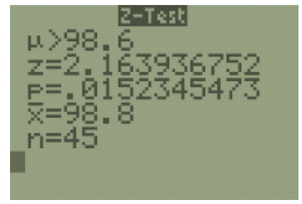


2*Normalcdf(2.504,1E99,0,1) = 0.0123

27. A sample of 45 body temperatures of athletes had a mean of 98.8˚F. Assume the population standard deviation is known to be 0.62˚F. Test the claim that the mean body temperature for all athletes is more than 98.6˚F. Use a 1% level of significance. Show all your steps using the p-value method.







H_0: $\mu \leq 98.6$
H_1: $\mu > 98.6$ (claim)

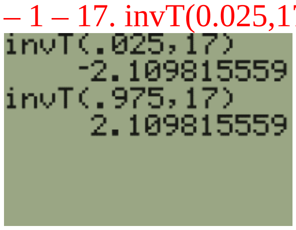
$z = \frac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \frac{98.8-98.6}{\left(\frac{0.62}{\sqrt{45}}\right)} = 2.1639$

p-value = 0.0152

The p-value is greater than $\alpha = 0.01$, therefore the decision is to not reject H_0.

Summary: There is not enough evidence to support the claim that the mean body temperature for all athletes is more than 98.6˚F

29. Compute the t critical value for a two-tailed test when α = 0.05 with a sample size of 18.
For the lower tail area use 0.05/2 = 0.025, for the upper tail area use 1 – 0.025 = 0.975, df = n – 1 – 17. invT(0.025,17) = -2.1098, invT(0.975,17) = 2.1098.



31. A student is interested in becoming an actuary. They know that becoming an actuary takes a lot of schooling and they will have to take out student loans. They want to make sure the starting salary will be higher than $55,000/year. They randomly sample 30 starting salaries for actuaries and find a p-value of 0.0392. Use α = 0.05.
a) Choose the correct hypotheses.
   i. H_0: $\mu = 55{,}000$ H_1: $\mu < 55{,}000$
   ii. H_0: $\mu > 55{,}000$ H_1: $\mu \leq 55{,}000$
   iii. H_0: $\mu = 55{,}000$ H_1: $\mu > 55{,}000$
   iv. H_0: $\mu < 55{,}000$ H_1: $\mu \geq 55{,}000$
   v. H_0: $\mu = 55{,}000$ H_1: $\mu \neq 55{,}000$

Answer: iii)
The student is testing that the starting salary is higher than $55,000, so we will reflect that in the alternative hypothesis with a > sign.

b) Should the student pursue an actuary career?

i. Yes, since we reject the null hypothesis.
ii. Yes, since we reject the claim.
iii. No, since we reject the claim.
iv. No, since we reject the null hypothesis.

Answer: i)

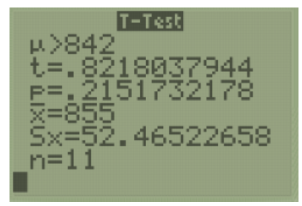
Since p < α, we reject the null hypothesis and conclude that the starting salary is higher than $55,000. Thus, the student should pursue the actuary career.

33. The average number of calories from a fast food meal for adults in the United States is 842 calories. A nutritionist believes that the average is higher than reported. They sample 11 meals that adults ordered and measure the calories for each meal shown below. Test the claim using a 5% level of significance. Assume that fast food calories are normally distributed. Show all 5 steps using the p-value method.

Calories	855	854	785	854	952	860	853	760	862	851	919

$H_0: \mu = 842$ $H_1: \mu > 842$

$t = \frac{855-842}{\left(52.465227/\sqrt{11}\right)} = 0.8218$ p-value = 0.2152



t-Test: Paired Two Sample for Means

	Calories	*Dummy*
Mean	855	0
Variance	2752.6	0
Observations	11	11
Pearson Correlation	#DIV/0!	
Hypothesized Mean Difference	842	
df	10	
t Stat	0.8218	
P(T<=t) one-tail	0.2152	
t Critical one-tail	1.8125	
P(T<=t) two-tail	0.4303	
t Critical two-tail	2.2281	

p-value = 0.2152 > α = 0.05 Do not reject H_0

We do not have evidence to support the claim the average calories from a fast food meal is higher than reported.

For exercises 35-39, show all 5 steps for hypothesis testing:

a) State the hypotheses.
b) Compute the test statistic.
c) Compute the critical value or p-value.
d) State the decision.
e) Write a summary.

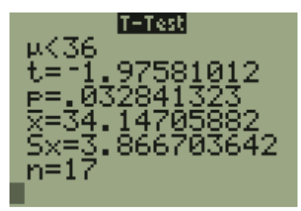
35. The total of individual pounds of garbage discarded by 17 households in one week is shown below. The current waste removal system company has a weekly maximum weight policy of 36 pounds. Test the claim that the average weekly household garbage weight is less than the company's weekly maximum. Use a 5% level of significance.

Weight				
34.5	32.9	42.9	32.9	31.8
40	33.8	35.8	35.4	30.5
31.4	39.2	26.8	30.6	34.5
34.7	32.8			

H_0: $\mu = 36$, H_1: $\mu < 36$

t-Test: Paired Two Sample for Means

	Weight	*Dummy*
Mean	34.1471	0
Variance	14.9514	0
Observations	17	17
Pearson Correlation	#DIV/0!	
Hypothesized Mean Difference	36	
df	16	
t Stat	-1.9758	
P(T<=t) one-tail	0.0328	
t Critical one-tail	1.7459	
P(T<=t) two-tail	0.0657	
t Critical two-tail	2.1199	



$t = \frac{34.1471-36}{\left({3.8667}/{\sqrt{17}}\right)} = -1.9758$ p-value = 0.0438

p-value = 0.0438 < α = 0.05 Reject H_0

There is enough evidence to support the claim the average weekly household garbage weight is less than the company's weekly 36 lb. maximum.

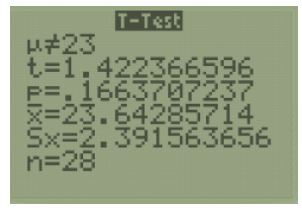
37. The average age of an adult's first vacation without a parent or guardian was reported to be 23 years old. A travel agent believes that the average age is different from what was reported. They sample 28 adults and they asked their age in years when they first vacationed as an adult without a parent or guardian, data shown below. Test the claim using a 10% level of significance. Show all 5 steps using the p-value method.

Age						
21	22	25	26	24	27	22
23	24	27	20	18	24	22
22	28	26	25	26	22	23
24	27	25	22	21	24	22

$H_0: \mu = 23$ $\quad$ $H_1: \mu \neq 23$

t-Test: Paired Two Sample for Means

	Age	*Dummy*
Mean	23.64286	0
Variance	5.719577	0
Observations	28	28
Pearson Correlation	#DIV/0!	
Hypothesized Mean Difference	23	
df	27	
t Stat	1.422367	
P(T<=t) one-tail	0.083185	
t Critical one-tail	1.313703	
P(T<=t) two-tail	0.166371	
t Critical two-tail	1.703288	



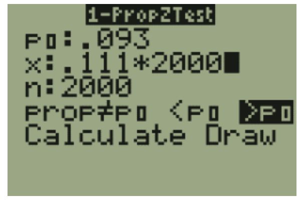
$t = \frac{23.642857-23}{\left(2.391564/\sqrt{28}\right)} = 1.4224$ $\quad$ p-value = 0.1664 $\quad$ p = 0.1664 > α = 0.10

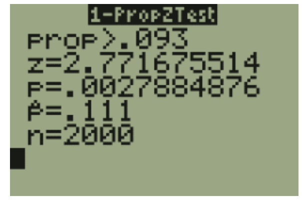
Do not reject H_0

We do not have evidence to support the claim that the mean age adults travel without a parent or guardian differs from 23.

39. The National Institute of Mental Health published an article stating that in any one-year period, approximately 9.3% of American adults suffer from depression or a depressive illness. Suppose that in a survey of 2000 people in a certain city, 11.1% of them suffered from depression or a depressive illness. Conduct a hypothesis test to determine if the true proportion of people in that city suffering from depression or a depressive illness is more than the 9.3% in the general adult American population. Test the relevant hypotheses using a 5% level of significance. Show all 5 steps using the p-value method.

$H_0: p = 0.093$
$H_1: p > 0.093$
$z = \frac{\hat{p} - p_0}{\sqrt{\left(\frac{p_0 q_0}{n}\right)}} = \frac{0.111 - 0.093}{\sqrt{\left(\frac{0.093 \cdot 0.907}{2000}\right)}} = 2.7116$
p-value = 0.0027
p-value = 0.0027 < α = 0.05   Reject H_0





There is enough evidence to support the claim the population proportion of American adults that suffer from depression or a depressive illness is more than 9.3%.

41. A 2019 survey by the Bureau of Labor Statistics reported that 92% of Americans working in large companies have paid leave. In January 2021, a random survey of workers showed that 89% had paid leave. The resulting p-value is 0.009; thus, the null hypothesis is rejected. It is concluded that there has been a decrease in the proportion of people, who have paid leave from 2019 to January 2021. What type of error is possible in this situation?
   a) Type I Error
   b) Type II Error
   c) Standard Error
   d) Margin of Error
   e) No error was made.

   Answer: a)
   A Type I Error is made when the null hypothesis is rejected incorrectly. Since the null hypothesis was rejected in this test, a Type I Error is a possibility.

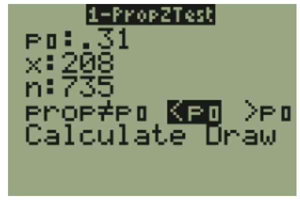
For exercises 43, show all 5 steps for hypothesis testing:

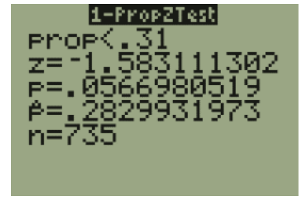
a) State the hypotheses.
b) Compute the test statistic.
c) Compute the critical value or p-value.
d) State the decision.
e) Write a summary.

43. You are conducting a study to see if the proportion of men over the age of 50 who regularly have their prostate examined is significantly less than 0.31. A random sample of 735 men over the age of 50 found that 208 have their prostate regularly examined. Do the sample data provide convincing evidence to support the claim? Test the relevant hypotheses using a 5% level of significance.

   $H_0: p = 0.31$
   $H_1: p < 0.31$

Before finding the test statistic, find the sample proportion $\hat{p} = \frac{208}{735}$ = 0.282993 and q_0 = 1 – 0.31 = 0.69.





$$z = \frac{\hat{p}-p_0}{\sqrt{\left(\frac{p_0 q_0}{n}\right)}} = \frac{0.282993-0.31}{\sqrt{\left(\frac{0.31 \cdot 0.69}{735}\right)}} = -1.5831$$

p-value = 0.0567

p-value = 0.0567 > α = 0.05
Fail to reject H_0

There is not enough evidence to support the claim the population proportion of men over the age of 50 who regularly have their prostate examined is significantly less than 0.31.

Chapter 9 Exercises

For exercises 1-5, show all 5 steps for hypothesis testing:

a) State the hypotheses.
b) Compute the test statistic.
c) Compute the critical value or p-value.
d) State the decision.
e) Write a summary.

1. An adviser is testing out a new online learning module for a placement test. Test the claim that on average the new online learning module increased placement scores at a significance level of $\alpha = 0.05$. For the context of this problem, $\mu_D = \mu_{Before} - \mu_{After}$ where the first data set represents the after test scores and the second data set represents before test scores. Assume the population is normally distributed. You obtain the following paired sample of 19 students that took the placement test before and after the learning module.

Before	After
55.8	57.1
51.7	58.3
76.6	83.6
47.5	49.5
48.6	51.1
11.4	20.6

Before	After
30.6	35.2
53	46.7
21	22.5
58.5	47.7
42.6	51.5
61.2	76.6

Before	After
26.8	28.6
11.4	14.5
56.3	43.7
46.1	57
72.8	66.1
42.2	38.1
51.3	42.4

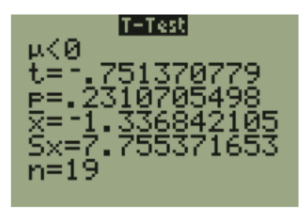
a) State the hypotheses. Before < After $H_0: \mu_D = 0$, $H_1: \mu_D < 0$

b) Compute the test statistic. $t = \frac{\bar{d} - \mu_d}{\left(s_d / \sqrt{n} \right)} = \frac{-1.3368 - 0}{\left(7.7553 / \sqrt{19} \right)} = -0.7514$

c) Compute the p-value.

t-Test: Paired Two Sample for Means

	Before	After
Mean	45.5474	46.8842
Variance	333.8804	327.577
Observations	19	19
Pearson Correlation	0.9091	
Hypothesized Mean Difference	0	
df	18	
t Stat	-0.7514	
P(T<=t) one-tail	0.2311	
t Critical one-tail	1.7341	
P(T<=t) two-tail	0.4621	
t Critical two-tail	2.1009	



p-value = 0.2311

d) State the decision. Do not reject H_0

e) Write a summary.

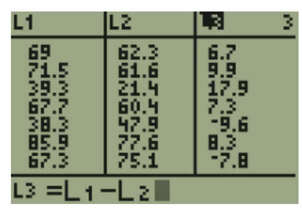
There is not enough evidence to support the claim on average the new online learning module increased placement scores.

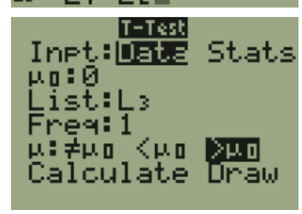
3. A manager wishes to see if the time (in minutes) it takes for their workers to complete a certain task will decrease when they are allowed to wear earbuds at work. A random sample of 20 workers' times was collected before and after. Test the claim that the time to complete the task has decreased at a significance level of $\alpha = 0.01$. For the context of this problem, $\mu_D = \mu_{before} - \mu_{after}$ where the first data set represents before measurement and the second data set represents the after measurement. Assume the population is normally distributed. You obtain the following sample data.

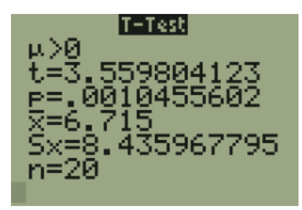
Before	After	Before	After
69	62.3	61.7	56.8
71.5	61.6	55.9	44.7
39.3	21.4	56.8	50.6
67.7	60.4	71	63.4
38.3	47.9	80.6	68.9
85.9	77.6	59.8	35.5
67.3	75.1	72.1	77
59.8	46.3	49.9	38.4
72.1	65	56.2	55.4
79	83	63.3	51.6

a) State the hypotheses. Before > After $H_0: \mu_D = 0, H_1: \mu_D > 0$

b) Compute the test statistic. $t = \frac{\bar{d}-\mu_d}{\left(s_d/\sqrt{n}\right)} = \frac{6.715-0}{\left(8.43597/\sqrt{20}\right)} = 3.5598$







t-Test: Paired Two Sample for Means

	Before	After
Mean	63.86	57.145
Variance	155.5036	243.7794
Observations	20	20
Pearson Correlation	0.842618	
Hypothesized Mean Difference	0	
df	19	
t Stat	3.5598	
P(T<=t) one-tail	0.0010	
t Critical one-tail	2.5395	
P(T<=t) two-tail	0.0021	
t Critical two-tail	2.8609	

c) Compute the p-value. p-value = 0.0010

d) State the decision. Reject H_0, since the p-value < α.

e) Write a summary. There is enough evidence to support the claim that the mean time to complete a task decreases when workers are allowed to wear their ear buds.

5. A researcher is testing reaction times between the dominant and non-dominant hand. They randomly start with different hands for 20 subjects and their reaction times for both hands is recorded in milliseconds. Use the following computer output to test to see if the reaction time is faster for the dominant hand using a 5% level of significance.

t-Test: Paired Two Sample for Means		
	Non-Dominant	*Dominant*
Mean	63.33	56.28
Variance	218.9643158	128.7522105
Observations	20	20
Pearson Correlation	0.9067	
Hypothesized Mean Difference	0	
df	19	
t Stat	4.7951	
P(T<=t) one-tail	0.0001	
t Critical one-tail	1.7291	
P(T<=t) two-tail	0.0001	
t Critical two-tail	2.0930	

a) State the hypotheses. Non-Dominant > Dominant H_0: $\mu_D = 0$, H_1: $\mu_D > 0$
b) Compute the test statistic. t Stat = t = 4.7951
c) Compute p-value. P(T<=t) one-tail = 0.0001
d) State the decision. Reject H_0, since the p-value < α.
e) Write a summary. There is enough evidence to support the claim that the mean reaction time is significantly faster for a person's dominant hand.

7. Doctors developed an intensive intervention program for obese patients with heart disease. Subjects with a BMI of 30 kg/m^2 or more with heart disease were assigned to a three-month lifestyle change of diet and exercise. Patients' Left Ventricle Ejection Fraction (LVEF) are measured before and after intervention. Assume that LVEF measurements are normally distributed.

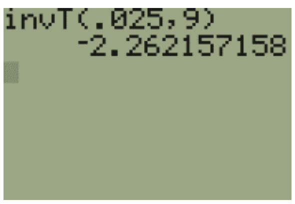
Before	After
44	56
49	58
50	64
49	60
57	63
62	71
39	49
41	51
52	60
42	55

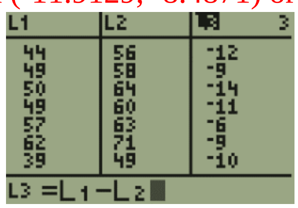
a) Find the 95% confidence interval for the mean of the differences.

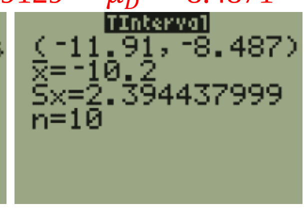
$\bar{D} \pm t_{\alpha/2}\left(\frac{s_D}{\sqrt{n}}\right)$ where $t_{\alpha/2}$= invT(0.025,9) = -2.262157

$-10.2 \pm 2.262157\left(\frac{2.394438}{\sqrt{10}}\right)$ -10.2 ± 1.782878

Use interval notation (-11.9129, -8.4871) or standard notation -11.9129 < μ_D < -8.4871







b) Using the confidence interval answer, did the intensive intervention program significantly increase the mean LVEF? Explain why.

Yes, since $\mu_D = 0$ is not captured in the interval (-11.9129, -8.4871).

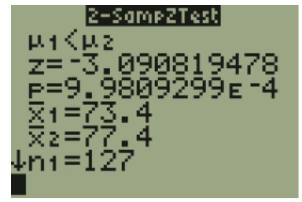
For exercises 9-13, show all 5 steps for hypothesis testing:

a) State the hypotheses.
b) Compute the test statistic.
c) Compute the critical value or p-value.
d) State the decision.
e) Write a summary.

9. A liberal arts college in New Hampshire implemented an online homework system for their introductory math courses and wanted to know whether the system improved test scores. In the Fall semester, homework was completed with pencil and paper, checking answers in the back of the book. In the Spring semester, homework was completed online – giving students instant feedback on their work. The results are summarized below. Population standard deviations were used from past studies. Is there evidence to suggest that the online system improves test scores? Use α = 0.05.

	Fall Semester	Spring Semester
Number of Students	127	144
Mean Test Score	73.4	77.4
Population Standard Deviation	10.2	11.1

Since the population standard deviations are known, use the 2-Sample Z-Test.
Fall Semester Score < Spring Semester Score



H_0: $\mu_1 = \mu_2$, H_1: $\mu_1 < \mu_2$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}} = \frac{(73.4 - 77.4) - 0}{\sqrt{\left(\frac{10.2^2}{127} + \frac{11.1^2}{144}\right)}} = -3.0908$$

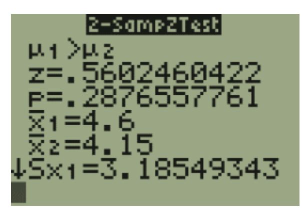
p-value = 0.0009981 Reject H_0, since the p-value < α.
There is enough evidence to support the claim that the online homework system for introductory math courses improved student's average test scores.

11. In Major League Baseball, the American League (AL) allows a designated hitter (DH) to bat in place of the pitcher, but in the National League (NL), the pitcher has to bat. However, when an AL team is the visiting team for a game against an NL team, the AL team must abide by the home team's rules and thus, the pitcher must bat. A researcher is curious if an AL team would score more runs for games in which the DH was used. She samples 20 games for an AL team for which the DH was used, and 20 games for which there was no DH. The data are below. Assume the population is normally distributed with a population standard deviation for runs scored of 2.54. Is there evidence to suggest that AL team would score more runs for games in which the DH was used? Use α = 0.10.

With Designated Hitter			
0	5	4	7
1	2	7	6
6	4	2	10
1	2	7	5
8	4	11	0

Without Designated Hitter			
3	6	5	2
12	4	0	1
6	3	7	8
4	0	5	1
2	4	6	4

Since the population standard deviations are known, use the 2-Sample Z-Test.
With DH > Without DH



H_0: $\mu_1 = \mu_2$, H_1: $\mu_1 > \mu_2$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}} = \frac{(4.6 - 4.15) - 0}{\sqrt{\left(\frac{2.54^2}{20} + \frac{2.54^2}{20}\right)}} = 0.5602$$

p-value = 0.2877 Do not reject H_0, since the p-value > α.
There is not enough evidence to support the claim that the American League team would score on average more runs for games in which the designated hitter was used.

13. A physical therapist believes that at 30 years old adults begin to decline in flexibility and agility. To test this, he randomly samples 35 of his patients who are less than 30 years old and 32 of his patients who are 30 or older and measure each patient's flexibility in the Sit-and-Reach test. The results are below. Is there evidence to suggest that adults under the age of 30 are more flexible? Use α = 0.05.

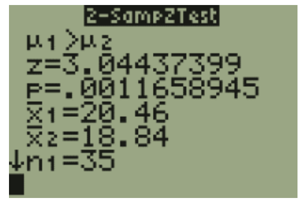
	Less Than 30	**30 or Older**
n	35	32
Mean Sit-and-Reach Score	20.46	18.84
Population Standard Dev.	2.237	2.118

Since the population standard deviations are known, use the 2-Sample Z-Test.

Less Than 30 > 30 or Older

H_0: $\mu_1 = \mu_2$, H_1: $\mu_1 > \mu_2$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}} = \frac{(20.46 - 18.84) - 0}{\sqrt{\left(\frac{2.237^2}{35} + \frac{2.118^2}{32}\right)}} = 3.0444$$



p-value = 0.0012 Reject H_0, since the p-value < α.

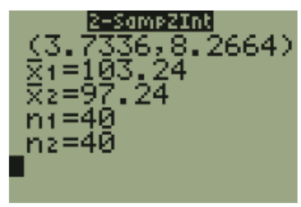
There is enough evidence to support the claim that adults under the age of 30 are more flexible.

15. A survey found that the average daily cost to rent a car in Los Angeles is \$103.24 and in Las Vegas is \$97.24. The data were collected from two random samples of 40 in each of the two cities and the population standard deviations are \$5.98 for Los Angeles and \$4.21 for Las Vegas. At the 0.05 level of significance, construct a confidence interval for the difference in the means and then decide if there is a significant difference in the rates between the two cities using the confidence interval method.

Since the population standard deviations are known, use the 2-Sample Z-Int.

H_0: $\mu_1 = \mu_2$, H_1: $\mu_1 \neq \mu_2$

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}$$



$$(103.24 - 97.24) \pm -1.95996\sqrt{\left(\frac{5.98^2}{40} + \frac{4.21^2}{40}\right)}$$

6 ± -2.266377 $3.7336 < \mu_1 - \mu_2 < 8.2664$

Since $\mu_1 - \mu_2 = 0$ is not between the endpoints of the confidence interval, we can reject H_0. There is a statistically significant difference in the mean daily car rental cost between Las Angeles and Las Vegas at the 5% level of significance.

For exercises 17-25, show all 5 steps for hypothesis testing:

a) State the hypotheses.
b) Compute the test statistic.
c) Compute the critical value or p-value.
d) State the decision.
e) Write a summary.

17. A movie theater company wants to see if there is a difference in the average movie ticket sales in San Diego and Portland per week. They sample 20 sales from San Diego and 20 sales from Portland over a week. Test the claim using a 5% level of significance. Assume the population variances are unequal, the samples are independent and that movie sales are normally distributed.

San Diego			
223	243	231	235
221	182	217	211
206	229	219	239
215	214	234	221
226	233	239	232

Portland			
233	228	209	214
219	212	214	222
226	216	223	220
226	219	221	223
219	211	218	224

H_0: $\mu_1 = \mu_2$, H_1: $\mu_1 \neq \mu_2$

t-Test: Two-Sample Assuming Unequal Variances		
	San Diego	*Portland*
Mean	223.5	219.85
Variance	199	37.0815789
Observations	20	20
Hypothesized Mean Difference	0	
df	26	
t Stat	1.0623725	
P(T<=t) one-tail	0.1489176	
t Critical one-tail	1.7056179	
P(T<=t) two-tail	0.2978352	
t Critical two-tail	2.0555294	

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} = \frac{223.5 - 219.85}{\sqrt{\left(\frac{199}{20} + \frac{37.0815789}{20}\right)}} = 1.0624$$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\left(\frac{s_1^2}{n_1}\right)^2 \left(\frac{1}{n_1 - 1}\right) + \left(\frac{s_2^2}{n_2}\right)^2 \left(\frac{1}{n_2 - 1}\right)\right)} = 26$$

p-value = 0.2978 Do not reject H_0

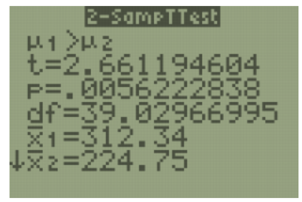
There is not enough evidence to support the claim that there is a difference in the average movie ticket sales in San Diego and Portland per week.

19. A national food product company believes that it sells more frozen pizza during the winter months than during the summer months. Weekly samples of sales found the following statistics in volume of sales (in hundreds of pounds). Use α = 0.10. Use the p-value method to test the company's claim. Assume the populations are approximately normally distributed with equal variances

Season	n	$\bar{x}$	s
Winter	24	312.34	135
Summer	22	224.75	84.42

H_0: $\mu_1 = \mu_2$, H_1: $\mu_1 > \mu_2$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} = \frac{312.34 - 224.75}{\sqrt{\left(\frac{135^2}{24} + \frac{84.42^2}{22}\right)}} = 2.6612$$



$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\left(\frac{s_1^2}{n_1}\right)^2 \left(\frac{1}{n_1 - 1}\right) + \left(\frac{s_2^2}{n_2}\right)^2 \left(\frac{1}{n_2 - 1}\right)\right)} = 39$$

p-value = 0.0056 Reject H_0

There is enough evidence to support the claim that the mean number of frozen pizzas sold during the winter months is more than during the summer months.

21. "Durable press" cotton fabrics are treated to improve their recovery from wrinkles after washing. "Wrinkle recovery angle" measures how well a fabric recovers from wrinkles. Higher scores are better. Here are data on the wrinkle recovery angle (in degrees) for a random sample of fabric specimens. Assume the populations are approximately normally distributed with unequal variances. A manufacturer believes that the mean wrinkle recovery angle for Hylite is better. A random sample of 20 Permafresh (group 1) and 25 Hylite (group 2) were measured. Test the claim using a 10% level of significance.

Permafresh			
124	139	164	142
144	102	131	118
136	127	137	148
117	137	147	129
133	137	148	135

Hylite				
139	146	139	139	146
131	138	138	132	142
133	142	138	137	134
146	137	138	138	133
139	140	141	140	141

H_0: $\mu_1 = \mu_2$, H_1: $\mu_1 < \mu_2$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} = \frac{134.75 - 138.68}{\sqrt{\left(\frac{180.197368}{20} + \frac{16.476667}{25}\right)}} = -1.2639$$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\left(\frac{s_1^2}{n_1}\right)^2 \left(\frac{1}{n_1 - 1}\right) + \left(\frac{s_2^2}{n_2}\right)^2 \left(\frac{1}{n_2 - 1}\right)\right)} = 22$$

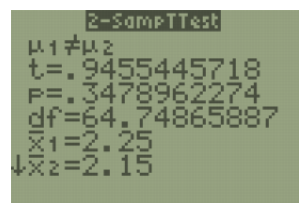
t-Test: Two-Sample Assuming Unequal Variances

	Permafresh	*Hylite*
Mean	134.75	138.68
Variance	180.1973684	16.47666667
Observations	20	25
Hypothesized Mean Difference	0	
df	22	
t Stat	-1.263872394	
P(T<=t) one-tail	0.109752065	
t Critical one-tail	1.321236742	
P(T<=t) two-tail	0.219504131	
t Critical two-tail	1.717144374	

p-value = 0.1098 Do not reject H_0
There is not enough evidence to support the claim that the mean wrinkle recovery angle for Hylite is better than Permafresh.

23. Two competing fast food restaurants advertise that they have the *fastest* wait time from when you order to when you receive your meal. A curious critic takes a random sample of 40 customers at each restaurant to test the claim. They find that Restaurant A has a sample mean wait time of 2.25 minutes with a standard deviation of 0.35 minutes and Restaurant B has a sample mean wait time of 2.15 minutes with a standard deviation of 0.57 minutes in wait time. Can they conclude that the mean wait time is significantly different for the two restaurants? Test at $\alpha = 0.05$. Assume the population variances are unequal.

H_0: $\mu_1 = \mu_2$, H_1: $\mu_1 \neq \mu_2$
t = 0.9455; p-value = 0.3479; Do not reject H_0; There is not enough evidence to support the claim that the mean wait time for the two restaurants is different.



25. A new over-the-counter medicine to treat a sore throat is to be tested for effectiveness. The makers of the medicine take two random samples of 25 individuals showing symptoms of a sore throat. Group 1 receives the new medicine and Group 2 receives a placebo. After a few days on the medicine, each group is interviewed and asked how they would rate their comfort level 1-10 (1 being the most uncomfortable and 10 being no discomfort at all). The results are below. Is there sufficient evidence to conclude the mean scores from Group 1 is more than Group 2? Test at $\alpha = 0.01$. Assume the populations are normally distributed and have unequal variances.

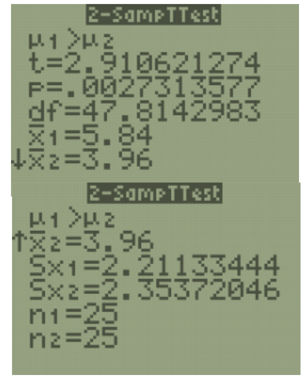
Group 1				
3	5	6	7	5
3	4	5	7	7
3	2	5	8	8
7	7	8	4	8
4	8	3	9	10

Group 2				
4	5	8	3	5
2	7	8	2	4
1	2	2	3	2
1	3	5	5	1
6	4	7	8	1

H_0: $\mu_1 = \mu_2$, H_1: $\mu_1 > \mu_2$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} = \frac{5.84 - 3.96}{\sqrt{\left(\frac{2.21133^2}{25} + \frac{2.35372^2}{25}\right)}} = 2.9106$$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\left(\left(\frac{s_1^2}{n_1}\right)^2\left(\frac{1}{n_1 - 1}\right) + \left(\frac{s_2^2}{n_2}\right)^2\left(\frac{1}{n_2 - 1}\right)\right)} = 47.814$$



p-value = 0.0027 Reject H_0

There is enough evidence to support the claim that the new medicine is effective.

27. A researcher takes sample temperatures in Fahrenheit of 17 days from New York City and 18 days from Phoenix. Test the claim that the mean temperature in New York City is different from the mean temperature in Phoenix. Use a significance level of $\alpha = 0.10$. Assume the populations are approximately normally distributed with unequal variances. You obtain the following two samples of data. Use the confidence interval method.

New York City		
98	85.4	87.7
95.5	75.4	86.1
92.2	79.5	74.3
102	82.4	85.2
85.4	64.3	82.8
80	65.5	

Phoenix		
106.8	82	120.1
98.6	72	114.4
91.5	115.2	93.7
82	94.2	89.7
97.7	72	104.7
64.9	86.8	76.6

Excel does not have a two-sample confidence interval, but we can get the descriptive statistics and critical value from Excel.

t-Test: Two-Sample Assuming Unequal Variances

	New York City	*Phoenix*
Mean	83.62941	92.38333
Variance	104.9535	252.6603
Observations	17	18
Hypothesized Mean Difference	0	
df	29	
t Stat	-1.94722	
P(T<=t) one-tail	0.030625	
t Critical one-tail	1.311434	
P(T<=t) two-tail	0.06125	
t Critical two-tail	1.699127	

H_0: $\mu_1 = \mu_2$, H_1: $\mu_1 \neq \mu_2$

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}$$

$$(83.62941 - 92.38333) \pm -1.69913\sqrt{\left(\frac{104.9535}{17} + \frac{252.6603}{18}\right)}$$

-8.75392 ± -7.638609

$-16.3925 < \mu_1 - \mu_2 < -1.1153$

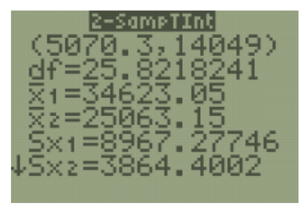
Since $\mu_1 - \mu_2 = 0$ is not between the endpoints of the confidence interval, we can reject H_0. There is enough evidence to support the claim that the mean temperature in New York City is different from the mean temperature in Phoenix.

29. Two random samples are taken from private and public universities (out-of-state tuition) around the nation. The yearly tuition is recorded from each sample and the results can be found below. Find the 95% confidence interval for the mean difference between private and public institutions. Assume the populations are normally distributed and have unequal variances.

Private Institutions			
43,120	34,750	29,498	30,129
28,190	44,897	31,980	33,980
34,490	32,198	22,764	47,909
20,893	18,432	54,190	32,200
42,984	33,981	37,756	38,120

Public Institutions			
25,469	21,871	22,650	28,745
19,450	24,120	29,143	30,120
18,347	27,450	25,379	21,190
28,560	29,100	23,450	21,540
32,592	21,870	23,871	26,346

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}$$



$$(34623.05 - 25063.15) \pm 2.056219949\sqrt{\left(\frac{80412065}{20} + \frac{14933588.87}{20}\right)}$$

9559.9 ± 4489.5728

$\$5,070.33 < \mu_1 - \mu_2 < \$14,049.47$

For exercises 31-33, show all 5 steps for hypothesis testing:

a) State the hypotheses.
b) Compute the test statistic.
c) Compute the critical value or p-value.
d) State the decision.
e) Write a summary.

31. A large shoe company is interested in knowing if the amount of money a customer is willing to pay on a pair of shoes is different depending on location. They take a random sample of 50 single-pair purchases from Southern states and another random sample of 50 single-pair purchases from Midwestern states and record the cost for each. The results can be found below. At the 0.05 level of significance, is there evidence that the mean cost differs between the Midwest and the South? Assume the population variances are equal.

Midwest (cost in dollars)				
70	43	21	62	45
60	23	15	37	66
65	38	30	46	64
71	54	51	61	82
33	79	28	84	63
68	72	78	43	84
78	80	24	80	16
82	45	38	84	84
73	23	36	69	78
76	71	38	46	18

South (cost in dollars)				
73	75	34	59	81
80	17	18	65	27
62	32	60	60	56
30	28	31	17	34
36	54	48	85	54
46	55	30	41	53
80	16	67	36	39
22	16	38	46	50
16	49	43	54	27
83	50	57	51	68

t-Test: Two-Sample Assuming Equal Variances

	Midwest	*South*
Mean	55.5	46.98
Variance	480.9489796	388.1832653
Observations	50	50
Pooled Variance	434.5661224	
Hypothesized Mean Difference	0	
df	98	
t Stat	2.043533049	
P(T<=t) one-tail	0.021841632	
t Critical one-tail	1.660551217	
P(T<=t) two-tail	0.043683264	
t Critical two-tail	1.984467455	

H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$; t = 2.0435; p-value = 0.0437; reject H_0; There is enough evidence to support the claim that the mean cost for a pair of shoes in the Midwest and the South are different.

33. The CEO of a large manufacturing company is curious if there is a difference in productivity level of her warehouse employees based on the region of the country the warehouse is located in. She randomly selects 35 employees who work in warehouses on the East Coast and 35 employees who work in warehouses in the Midwest and records the number of parts shipped out from each for a week. She finds that East Coast group ships an average of 1,287 parts and a standard deviation of 348. The Midwest group ships an average of 1,449 parts and a standard deviation of 298. Using a 0.01 level of significance, test if there is a difference in productivity level. Assume the population variances are equal.

Since the population variances are unknown and equal, use the pooled 2-Sample T-Test.
H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 \neq \mu_2$

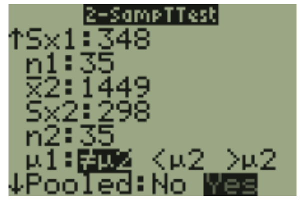
Test Statistic is $t = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)_0}{\sqrt{\left(\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{(n_1+n_2-2)}\right)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{(1287-1449)}{\sqrt{\left(\left(\frac{(34\cdot 348^2+34\cdot 298^2)}{68}\right)\cdot\left(\left(\frac{1}{35}\right)+\left(\frac{1}{35}\right)\right)\right)}} = -2.0919$

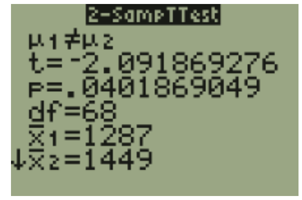
Use the t-distribution with pooled degrees of freedom $df = n_1 + n_2 - 2 = 68$.

p-value = 0.0402

Do not reject H_0, since the p-value > α.

There is not enough evidence to support the claim that there is a statistically significant difference in the mean productivity level between the two locations.





35. A pet store owner believes that dog owners, on average spend a different amount on their pets compared to cat owners. The owner randomly records the sales of 40 customers who said they only owned dogs and found the mean of the sales of \$56.07 with a standard deviation of \$24.50. The owner randomly records the sales of 40 customers who said they only owned cats and found a mean of the sales of \$52.92 with a standard deviation of \$23.53. Find the 95% confidence interval to test the pet store owner's claim. Assume the population variances are equal.

H_0: $\mu_1 = \mu_2$, H_1: $\mu_1 \neq \mu_2$

$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\left(\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{(n_1+n_2-2)}\right)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

$(56.07 - 52.92) \pm -1.990847\sqrt{\left(\frac{39*24.5^2+39*23.53^2}{78}\right)\left(\frac{1}{40}+\frac{1}{40}\right)}$

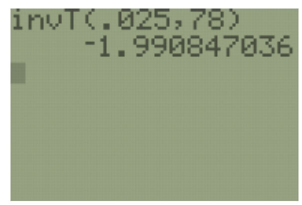
3.15 ± -10.692864

$-7.5429 < \mu_1 - \mu_2 < 13.8429$

Since $\mu_1 - \mu_2 = 0$ is between the endpoints of the confidence interval, we fail to reject H_0.

Do not reject H_0.

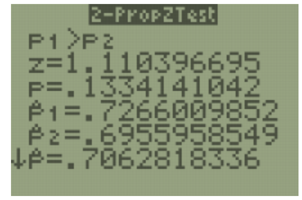
There is not enough evidence to support the claim that dog owners spend more on average than cat owners on their pets.



For exercises 37-41, show all 5 steps for hypothesis testing:

a) State the hypotheses.
b) Compute the test statistic.
c) Compute the critical value or p-value.
d) State the decision.
e) Write a summary.

37. A random sample of 406 college freshman found that 295 bought most of their textbooks from the college's bookstore. A random sample of 772 college seniors found that 537 bought their textbooks from the college's bookstore. You wish to test the claim that the proportion of all freshman that purchase most of their textbooks from the college's bookstore is greater than the proportion of all seniors at a significance level of α = 0.01.

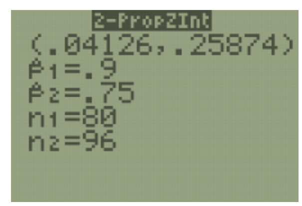


H_0: $p_1 = p_2$, H_1: $p_1 > p_2$
z = 1.1104; p-value = 0.1334; fail to reject H_0; There is not enough evidence to support the claim that the proportion of all freshman that purchase most of their textbooks from the college's bookstore is greater than the proportion of all seniors.

39. TDaP is a booster shot that prevents Diphtheria, Tetanus, and Pertussis in adults and adolescents. The shot should be administered every 8 years in order for it to remain effective. A random sample of 500 people living in a town that experienced a pertussis outbreak this year were divided into two groups. Group 1 was made up of 132 individuals who had not had the TDaP booster in the past 8 years, and Group 2 consisted of 368 individuals who had. In Group 1, 15 individuals caught pertussis during the outbreak, and in Group 2, 11 individuals caught pertussis. Is there evidence to suggest that the proportion of individuals who caught pertussis and were not up to date on their booster shot is significantly higher than those that were? Test at the 0.05 level of significance.

H_0: $p_1 = p_2$, H_1: $p_1 > p_2$
Test Statistic z = 3.717742
p-value = 0.00010051
Reject H_0. Yes, there is evidence that the proportion of those who caught pertussis is higher for those who were not up to date on their booster.

41. In a sample of 80 faculty from Portland State University, it was found that 90% were union members, while in a sample of 96 faculty at University of Oregon, 75% were union members. Find the 95% confidence interval for the difference in the proportions of faculty that belong to the union for the two universities.



$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\left(\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}\right)}$$
$$(0.9 - 0.75) \pm 1.96\sqrt{\left(\frac{0.9 \cdot 0.1}{80} + \frac{0.75 \cdot 0.25}{96}\right)}$$
$$0.04126 < p_1 - p_2 < 0.25874$$

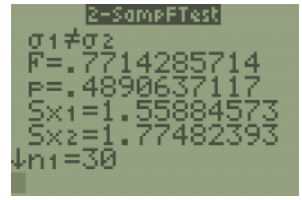
43. What is the critical value for a right-tailed F-test with a 5% level of significance with $df_1 = 4$ and $df_2 = 33$? Round answer to 4 decimal places. =F.INV.RT(0.05,4,33) = 2.6589

45. What is the critical value for a left-tailed *F*-test with a 10% level of significance with $df_1 = 29$ and $df_2 = 20$? Round answer to 4 decimal places. =F.INV(0.1,29,20) = 0.5967

For exercises 47-61, show all 5 steps for hypothesis testing:

a) State the hypotheses.
b) Compute the test statistic.
c) Compute the critical value or p-value.
d) State the decision.
e) Write a summary.

47. A researcher wants to compare the variances of the heights (in inches) of four-year college basketball players with those of players in junior colleges. A sample of 30 players from each type of school is selected, and the variances of the heights for each type are 2.43 and 3.15 respectively. At α = 0.10, test to see if there a significant difference between the variances of the heights in the two types of schools.



$H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 \neq \sigma_2^2$

$F = \frac{s_1^2}{s_2^2} = \frac{2.43}{3.15} = 0.7714$ p-value = 0.4891

Fail to reject H_0
There is not enough evidence to support the claim that there a significant difference between the variances of the heights of four-year college basketball players with those of players in junior colleges.

49. A researcher takes sample temperatures in Fahrenheit of 17 days from New York City and 18 days from Phoenix. Test the claim that the standard deviation of temperatures in New York City is different from the standard deviation of temperatures in Phoenix. Use a significance level of α = 0.05. Assume the populations are approximately normally distributed. You obtain the following two samples of data.

New York City		
98	85.4	87.7
95.5	75.4	86.1
92.2	79.5	74.3
102	82.4	85.2
85.4	64.3	82.8
80	65.5	

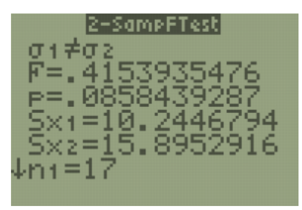
Phoenix		
106.8	82	120.1
98.6	72	114.4
91.5	115.2	93.7
82	94.2	89.7
97.7	72	104.7
64.9	86.8	76.6

$H_0: \sigma_1 = \sigma_2$
$H_1: \sigma_1 \neq \sigma_2$

$F = \frac{s_1^2}{s_2^2} = 0.4154$

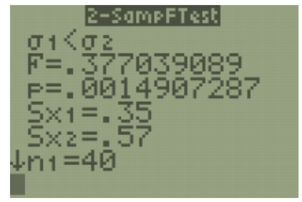
F-Test Two-Sample for Variances

	New York City	*Phoenix*
Mean	83.62941176	92.38333333
Variance	104.9534559	252.6602941
Observations	17	18
df	16	17
F	0.415393547	
P(F<=f) one-tail	0.042921964	
F Critical one-tail	0.365230351	



CV = 0.3652 & 2.6968 or p-value = 0.042921964*2 = 0.0858; Fail to reject H_0; There is not enough evidence to support the claim that there a significant difference between the standard deviation of temperatures in New York City compared to Phoenix.

51. Two competing fast food restaurants advertise that they have the *fastest* wait time from when you order to when you receive your meal. A curious critic takes a random sample of 40 customers at each restaurant and finds that there is no statistically significant difference in the average wait time between the two restaurants. Both restaurants are in fact advertising truthfully then. However, as a skeptical statistician, this critic knows that a high standard deviation may also keep a customer waiting for a long time on any given trip to the restaurant, so they test for the difference in standard deviation of wait time from this same sample. They find that Restaurant A has a sample standard deviation of 0.35 minutes and Restaurant B has a sample standard deviation of 0.57 minutes in wait time. Can they conclude that the standard deviation in wait time is significantly longer for Restaurant B? Test at $\alpha = 0.05$.



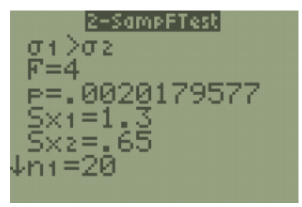
$H_0: \sigma_1 = \sigma_2$ $\quad F = \frac{s_1^2}{s_2^2} = \frac{0.35^2}{0.57^2} = 0.3770$ $\quad$ p-value = 0.0015

$H_1: \sigma_1 < \sigma_2$

Reject H_0.

There is enough evidence to support the claim that the standard deviation of wait times for Restaurant B is significantly longer than Restaurant A.

53. The manager at a pizza place has been getting complaints that the auto-fill soda machine is either under filling or over filling their cups. The manager ran several tests on the machine before using it and knows that the average fill quantity is 12 oz. – exactly as she was hoping. However, she did not test the variance. She took a random sample of 20 fills from her machine, and a random sample of 20 fills from another branch of the restaurant that has not been having complaints. From her machine, she found a sample standard deviation of 1.3 oz. and from the other restaurant's machine she found a sample standard deviation of 0.65 oz. At the 0.05 level of significance, does it seem her machine has a higher variance than the other machine? Assume the populations are normally distributed.



H_0: $\sigma_1^2 = \sigma_2^2$; H_1: $\sigma_1^2 > \sigma_2^2$; F = 4; p-value = 0.0020; Reject H_0;
There is enough evidence to support the claim that the soda machine has a higher variance compared to the other restaurant.

55. In a random sample of 100 college students, 47 were sophomores and 53 were seniors. The sophomores reported spending an average of $37.03 per week going out for food and drinks with a standard deviation of $7.23, while the seniors reported spending an average of $52.94 per week going out for food and drinks with a standard deviation of $12.33. Can it be concluded that there is a difference in the standard deviation spent on food and drinks between sophomores and seniors? Test at $\alpha = 0.10$.

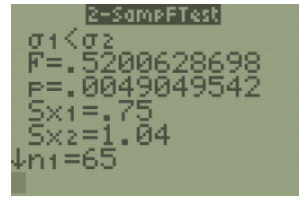
$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

F = 0.343835; p-value = 0.000336

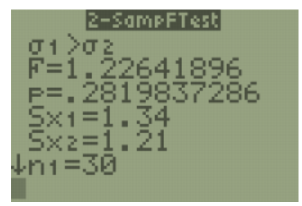
Reject H_0. There is evidence to claim that the standard deviation in money spent on food and drinks differs between sophomores and seniors.

57. The math department chair at a university is proud to boast an average satisfaction score of 8.4 out of 10 for her department's courses. This year, the English department advertised an average of 8.5 out of 10. Not to be outdone, The math department chair decides to check if there is a difference in how the scores vary between the departments. She takes a random sample of 65 math department evaluations and finds a sample standard deviation of 0.75 and a random sample of 65 English department evaluations and finds a sample standard deviation of 1.04. Does she have sufficient evidence to claim that the English department may have a higher average, but also has a higher standard deviation – meaning that their scores are not as consistent as the math department's? Test at $\alpha = 0.05$.



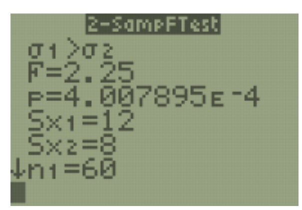
H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 < \sigma_2$; F = 0.5201; p-value = 0.0049; reject H_0; There is evidence to support the claim that the standard deviation in satisfaction scores is higher for the English department compared to the Math department.

59. The manager at a local coffee shop is trying to decrease the time customers wait for their orders. He wants to find out if keeping multiple registers open will make a difference. He takes a random sample of 30 customers when only one register is open and finds that they wait an average of 6.4 minutes to reach the front with a standard deviation of 1.34 minutes. He takes another random sample of 35 customers when two registers are open and finds that they wait an average of 4.2 minutes to reach the front with a standard deviation of 1.21 minutes. He takes both his samples during peak hours to maintain consistency. Can it be concluded at the 0.05 level of significance that there is a smaller standard deviation in wait time with two registers open?



H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 > \sigma_2$; F = 1.2264; p-value = 0.282; fail to reject H_0; There is evidence to claim that the standard deviation in wait time with two registers open is smaller.

61. In a random sample of 60 pregnant women with preeclampsia, their systolic blood pressure was taken right before beginning to push during labor. The mean systolic blood pressure was 174 with a standard deviation of 12. In another random sample of 80 pregnant women without preeclampsia, there was a mean systolic blood pressure of 133 and a standard deviation of 8 when the blood pressure was also taken right before beginning to push. Is there sufficient evidence to conclude that women with preeclampsia have a larger variation in blood pressure in the late stages of labor? Test at the 0.01 level of significance.



H_0: $\sigma_1 = \sigma_2$; H_1: $\sigma_1 > \sigma_2$; F = 2.25; p-value = 0.0004; reject H_0; There is evidence to claim that the standard deviation in blood pressure for women with preeclampsia has a larger variation in the late stages of labor.

Chapter 10 Exercises

1. The shape of the χ^2 Distribution is usually: Answer d)
   a) Normal
   b) Bell-shaped
   c) Skewed Left
   d) Skewed Right
   e) Uniform

3. What are the requirements to be satisfied before using a Goodness of Fit test? Check all that apply. Answers b and c
   a) The data are obtained using systematic sampling.
   b) The data are obtained from a simple random sample.
   c) The expected frequency from each category is 5 or more.
   d) The observed frequency from each category is organized from largest to smallest.
   e) The degrees of freedom are less than 30.

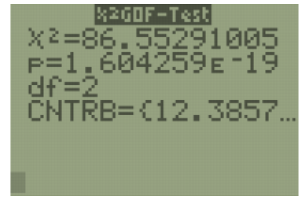
For exercises 5-19, show all 5 steps for hypothesis testing:

a) State the hypotheses.
b) Compute the test statistic.
c) Compute the critical value or p-value.
d) State the decision.
e) Write a summary.

5. A professor using an open-source introductory statistics book predicts that 60% of the students will purchase a hard copy of the book, 25% will print it out from the web, and 15% will read it online. At the end of the term she asks her students to complete a survey where they indicate what format of the book they used. Of the 126 students, 45 said they bought a hard copy of the book, 25 said they printed it out from the web, and 56 said they read it online. Run a Goodness of Fit test at α = 0.05 to see if the distribution is different than expected.

Format	**HC**	**Print**	**Online**	**Total**
Observed	45	25	56	126
Expected	126*0.6 = 75.6	126*0.25 = 31.5	126*0.15 = 18.9	126
$\frac{(O-E)^2}{E}$	$\frac{(45-75.6)^2}{75.6}$ = 12.3857	$\frac{(25-31.5)^2}{31.5}$ = 1.3413	$\frac{(56-18.9)^2}{18.9}$ = 72.8259	86.5529

H_0: $p_1 = 0.6$, $p_2 = 0.25$, $p_3 = 0.15$
H_1: At least one proportion is different.
Test Statistic is $\chi^2 = \sum \frac{(O-E)^2}{E} = 86.5529$; p-value = 1.604E-19 = 0
Reject H_0. There is enough evidence to support the claim that the distribution is different than expected. There were more students than expected that would read the text online.



7. You might think that if you looked at the first digit in randomly selected numbers that the distribution would be uniform. Actually, it is not! Simon Newcomb and later Frank Benford both discovered that the digits occur according to the following distribution.

Digit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Probability	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

A forensic accountant can use Benford's Law to detect fraudulent tax data. Suppose you work for the IRS and are investigating an individual suspected of embezzling. The first digit of 192 checks to a supposed company are as follows.

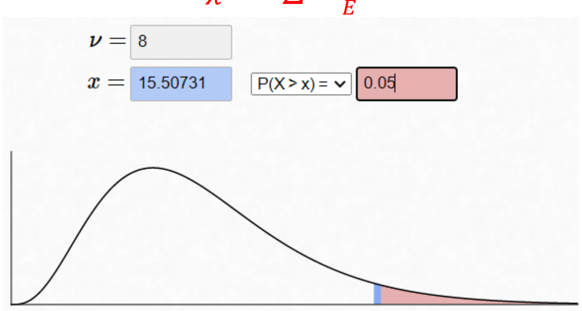
Digit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Observed Frequency	56	23	19	20	16	19	17	10	12

Run a complete Goodness of Fit test to see if the individual is likely to have committed tax fraud. Use $\alpha = 0.05$. Should law enforcement officials pursue the case? Explain.

H_0: $p_1 = 0.301$, $p_2 = 0.176$, $p_3 = 0.125$, $p_4 = 0.097$, $p_5 = 0.079$, $p_6 = 0.067$, $p_7 = 0.058$, $p_8 = 0.051$, $p_9 = 0.046$
H_1: At least one proportion is different.

Digit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Total**
O	56	23	19	20	16	19	17	10	12	192
E	57.792	33.792	24	18.624	15.168	12.864	11.136	9.792	8.832	192
$\frac{(O-E)^2}{E}$	0.0556	3.4466	1.0417	0.1017	0.0456	2.9268	3.0879	0.0044	1.1363	11.8466

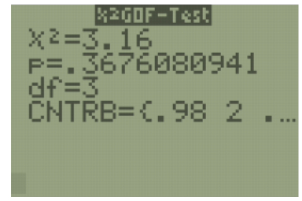
Test Statistic is $\chi^2 = \sum \frac{(O-E)^2}{E} = 11.8466$ CV = CHISQ.INV.RT(0.05,8) = 15.5073



Do not reject H_0. There is no evidence of tax fraud so law enforcement officials should not pursue the case.

9. Consumer panel preferences for four store displays follow. Test to see whether there is a preference among the four display designs. Use α = 0.05.

Display	A	B	C	D	Total
Observed	43	60	47	50	200
Expected	0.25*200 = 50	50	50	50	200
$\frac{(O-E)^2}{E}$	0.98	2	0.18	0	3.16



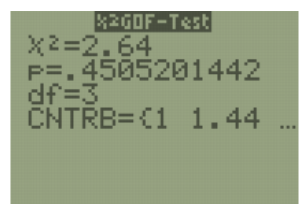
H_0: $p_1 = 0.25$, $p_2 = 0.25$, $p_3 = 0.25$, $p_4 = 0.25$
H_1: At least one proportion is different.

Test Statistic is $\chi^2 = \sum\frac{(O-E)^2}{E} = 3.16$ p-value = 0.3676

Do not reject H_0. There is not enough evidence to support the claim that preference among the four display designs.

11. The director of a Driver's Ed program is curious if the time of year has an impact on number of car accidents in the United States. They assume that weather may have a significant impact on the ability of drivers to control their vehicles. They take a random sample of 100 car accidents and record the season each occurred in. They found that 20 occurred in the spring, 31 in the summer, 23 in the fall, and 26 in the winter. Can it be concluded at the 0.05 level of significance that car accidents are not equally distributed throughout the year?

Season	Spring	Summer	Fall	Winter	Total
Observed	20	31	23	26	200
Expected	0.25*100 = 25	25	25	25	100
$\frac{(O-E)^2}{E}$	1	1.44	0.16	0.04	3.16



H_0: $p_1 = 0.25$, $p_2 = 0.25$, $p_3 = 0.25$, $p_4 = 0.25$
H_1: At least one proportion is different.

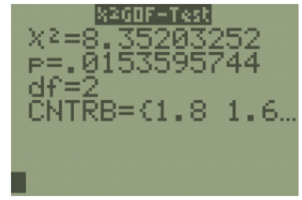
Test Statistic is $\chi^2 = \sum\frac{(O-E)^2}{E} = 2.64$ p-value = 0.4505

Do not reject H_0. There is not enough evidence to support the claim that car accidents are not equally distributed throughout the year.

13. The permanent residence of adults aged 18-25 in the U.S. was examined in a survey from the year 2000. The survey revealed that 27% of these adults lived alone, 32% lived with a roommate(s), and 41% lived with their parents/guardians. In 2008, during an economic recession in the country, another such survey of 1,500 people revealed that 378 lived alone, 452 lived with a roommate(s), and 670 lived with their parents. Is there a significant difference in where young adults lived in 2000 versus 2008? Test with a Goodness of Fit test at α = 0.05.

Residence	Alone	Roommate	Parent	Total
Observed	378	452	670	1500
Expected	1500*0.27 = 405	1500*0.32 = 480	1500*0.41 = 615	1500
$\frac{(O-E)^2}{E}$	1.8	1.63333	4.91869	8.352

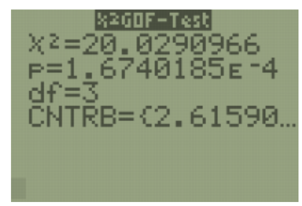
H_0: $p_1 = 0.27$, $p_2 = 0.32$, $p_3 = 0.41$
H_1: At least one proportion is different.
Test Statistic is $\chi^2 = \sum \frac{(O-E)^2}{E} = 8.352$ p-value = 0.0154
Reject H_0. There is enough evidence to support the claim that there a significant difference in where young adults lived in 2000 versus 2008. There are fewer young adults living at home than expected.



15. An urban economist is curious if the distribution in where Oregon residents live is different today than it was in 1990. She observes that today there are approximately 3,050 thousand residents in NW Oregon, 907 thousand residents in SW Oregon, 257 thousand in Central Oregon, and 106 thousand in Eastern Oregon. She knows that in 1990 the breakdown was as follows: 72.7% NW Oregon, 19.7% SW Oregon, 4.8% Central Oregon, and 2.8% Eastern Oregon. Can she conclude that the distribution in residence is different today at a 0.05 level of significance?

Area	**NW**	**SW**	**C**	**E**	**Total**
Observed	3050	907	257	106	4320
Expected	0.727*4320 = 3140.64	0.207*4320 = 851.04	0.048*4320 = 207.36	0.028*4320 = 120.96	4320
$\frac{(O-E)^2}{E}$	2.6159	3.6796	11.8833	1.8502	20.0291

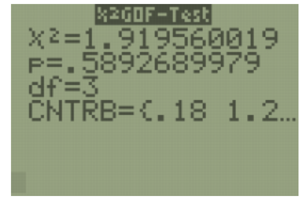
H_0: $p_1 = 0.727$, $p_2 = 0.197$, $p_3 = 0.048$, $p_4 = 0.028$
H_1: At least one proportion is different.
Test Statistic is $\chi^2 = \sum \frac{(O-E)^2}{E} = 20.0291$; p-value = 0.0002
Reject H_0. There is enough evidence to support the claim that the distribution of Oregon residents is different now compared to 1990. There were more Oregonians in central Oregon than expected.



17. Students at a high school are asked to evaluate their experience in a class at the end of each school year. The courses are evaluated on a 1-4 scale – with 4 being the best experience possible. In the History Department, the courses typically are evaluated at 10% 1's, 15% 2's, 34% 3's, and 41% 4's. A new history teacher, Mr. Mendoza, sets a goal to outscore these numbers. At the end of the year, he takes a random sample of his evaluations and finds 11 1's, 14 2's, 47 3's, and 53 4's. At the 0.05 level of significance, can Mr. Mendoza claim that his evaluations are significantly different from the History Department's?

Score	**1**	**2**	**3**	**4**	**Total**
Observed	11	14	47	53	125
Expected	0.1*125 = 12.5	0.15*125 = 18.75	0.34*125 = 42.5	0.41*125 = 51.25	125
$\frac{(O-E)^2}{E}$	0.18	1.20333	0.47647	0.05975	1.9196

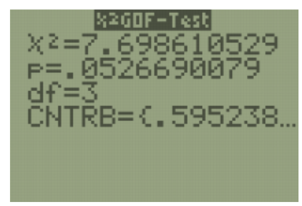
H_0: $p_1 = 0.1$, $p_2 = 0.15$, $p_3 = 0.34$, $p_4 = 0.41$
H_1: At least one proportion is different.
Test Statistic is $\chi^2 = \sum \frac{(O-E)^2}{E}$ = 1.9196 p-value = 0.5893
Do not reject H_0. There is not enough evidence to support the claim that the Mr. Mendoza's course evaluation scores are different compared to the rest of the History Department's evaluations.



19. A company that develops over-the-counter medicines is working on a new product that is meant to shorten the length of sore throats. To test their product for effectiveness, they take a random sample of 100 people and record how long it took for their symptoms to completely disappear. The results are in the table below. The company knows that on average (without medication) it takes a sore throat 6 days or less to heal 42% of the time, 7-9 days 31% of the time, 10-12 days 16% of the time, and 13 days or more 11% of the time. Can it be concluded at the 0.01 level of significance that the patients who took the medicine healed at a different rate than these percentages?

	6 days or less	**7-9 days**	**10-12 days**	**13 or more days**	**Total**
Duration of Sore Throat	47	38	10	5	100
Expected	42	31	16	11	100
$\frac{(O-E)^2}{E}$	0.5952	1.5806	2.25	3.2727	7.6986

H_0: $p_1 = 0.42$, $p_2 = 0.31$, $p_3 = 0.16$, $p_4 = 0.11$
H_1: At least one proportion is different.
Test Statistic is $\chi^2 = \sum \frac{(O-E)^2}{E}$ = 7.6986 p-value = 0.05267
Do not reject H_0. There is not enough evidence to support the claim that the patients who took the medicine healed at a different rate than these percentages.



21. The null hypothesis for the χ^2 Independence Test always states that _______________.
   a) The two values are equal.
   b) One variable is dependent on another variable.
   c) One variable is independent of another variable.
   d) The expected values and observed values are the same.

   Answer: c

For exercises 23-33, show all 5 steps for hypothesis testing:
a) State the hypotheses.
b) Compute the test statistic.
c) Compute the critical value or p-value.
d) State the decision.
e) Write a summary.

23. A manufacturing company knows that their machines produce parts that are defective on occasion. They have 4 machines producing parts and want to test if defective parts are dependent on the machine that produced it. They take a random sample of 300 parts and find the following results. Test at the 0.05 level of significance.

Observed	Machine 1	Machine 2	Machine 3	Machine 4	Total
Defective	9	12	15	6	42
Non-Defective	63	70	68	57	258
Total	72	82	83	63	300

Find the row and column totals. Compute the expected values by taking $\frac{\text{Row Total}\cdot\text{Column Total}}{\text{Grand Total}}$ for each of the 8 cells.

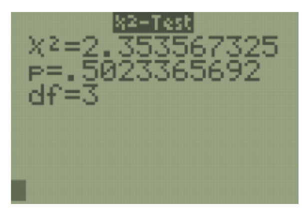
Expected	Machine 1	Machine 2	Machine 3	Machine 4	Total
Defective	10.08	11.48	11.62	8.82	42
Non-Defective	61.92	70.52	71.38	54.18	258
Total	72	82	83	63	300

H_0: The number of defective parts is independent on the machine that produced it.
H_1: The number of defective parts is dependent on the machine that produced it.

Compute the test statistic.

$\frac{(O-E)^2}{E}$	Machine 1	Machine 2	Machine 3	Machine 4	Total
Defective	$\frac{(9-10.08)^2}{10.08} =$ 0.1157	$\frac{(12-11.48)^2}{11.48} =$ 0.0236	$\frac{(15-11.62)^2}{11.62} =$ 0.9832	$\frac{(6-8.82)^2}{8.82} =$ 0.9016	
Non-Defective	$\frac{(63-61.92)^2}{61.92} =$ 0.0188	$\frac{(70-70.52)^2}{70.52} =$ 0.0038	$\frac{(68-71.38)^2}{71.38} =$ 0.1601	$\frac{(57-54.18)^2}{54.18} =$ 0.1468	2.3536

Test Statistic is $\chi^2 = \sum \frac{(O-E)^2}{E} = 2.3536$ p-value = 0.5023



Do not reject H_0. There is not enough evidence to support the claim that the number of defective parts is dependent on the machine that produced it.

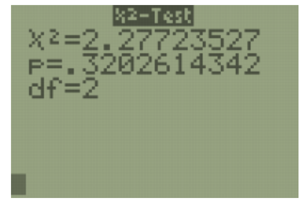
25. A restaurant chain that has 3 locations in Portland is trying to determine which of their 3 locations they should keep open on New Year's Eve. They survey a random sample of customers at each location and ask each whether they plan to go out to eat on New Year's Eve. The results are below. Run a test for independence to decide if the proportion of customers who will go out to eat on New Year's Eve is dependent on location. Use α = 0.05.

Observed	NW Location	NE Location	SE Location	Total
Will Go Out	45	33	36	114
Won't Go Out	23	29	25	77
Total	68	62	61	191

Expected	NW Location	NE Location	SE Location	Total
Will Go Out	40.5864	37.0052	36.4084	114
Won't Go Out	27.4136	24.9948	24.5916	77
Total	68	62	61	191

H_0: The proportion of customers who will go out to eat on New Year's Eve is independent of location.
H_1: The proportion of customers who will go out to eat on New Year's Eve is dependent on location.

Test Statistic is $\chi^2 = \sum \frac{(O-E)^2}{E}$ = 2.2772; p-value = 0.3203



Do not reject H_0.
There is not enough evidence to support the claim that the proportion of customers who will go out to eat on New Year's Eve is dependent on location.

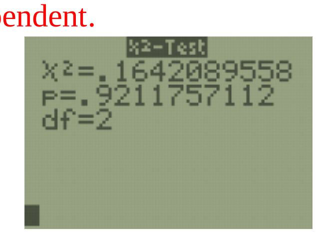
27. A high school offers math placement exams for incoming freshmen to place students into the appropriate math class during their freshman year. Three middle schools were sampled and the following pass/fail results were found. Test to see if the math placement exam and where students are placed are dependent at the 0.10 level of significance.

Observed	School A	School B	School C	Total
Pass	42	29	45	116
Fail	57	35	61	153
Total	99	64	106	269

Expected	School A	School B	School C	Total
Pass	42.69145	27.59851	45.71004	116
Fail	56.30855	36.401489	60.28996	153
Total	99	64	106	269

H_0: The math placement exam and where students are placed are independent.
H_1: The math placement exam and where students are placed are dependent.

Test Statistic is $\chi^2 = \sum \frac{(O-E)^2}{E}$ = 0.1642; p-value = 0.9212



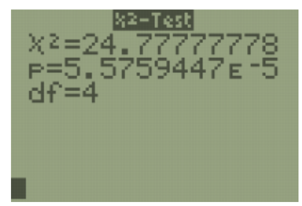
Do not reject H_0.
There is not enough evidence to support the claim that the math placement exam and where students are placed are dependent.

29. A university changed to a new learning management system (LMS) during the past school year. The school wants to find out how it is working for the different departments – the results in preference found from a survey are below. Test to see if the department and LMS preference are dependent at $\alpha = 0.05$.

Observed	Prefers Old LMS	Prefers New LMS	No Preference	Total
School of Business	15	24	6	45
College of Liberal Arts & Science	34	7	19	60
College of Education	21	19	5	45
Total	70	50	30	150

Expected	Prefers Old LMS	Prefers New LMS	No Preference	Total
School of Business	21	15	9	45
College of Liberal Arts & Science	28	20	12	60
College of Education	21	15	9	45
Total	70	50	30	133

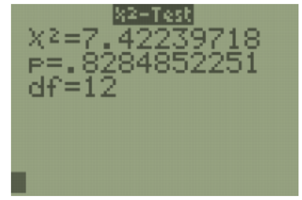
H_0: Department and LMS preference are independent.
H_1: Department and LMS preference are dependent.
Test Statistic is $\chi^2 = \sum \frac{(O-E)^2}{E}$ = 24.7778; p-value = 0.000056
Reject H_0. There is enough evidence to support the claim that department and LMS preference are dependent.



31. An electronics store has 4 branches in a large city. They are curious if sales in any particular department are different depending on location. They take a random sample of purchases throughout the 4 branches – the results are recorded below. Test to see if the type of electronic device and store branch are dependent at the 0.05 level of significance.

Observed	Appliances	TV	Computers	Cameras	Cellphones	Total
Branch 1	54	28	61	24	81	248
Branch 2	44	21	55	23	92	235
Branch 3	49	18	49	30	72	218
Branch 4	51	29	65	29	102	276
Total	198	96	230	106	347	977

Observed	App.	TV	Comp.	Cameras	Cellphones	Total
Branch 1	50.25998	24.36847	58.3828	26.90686	88.08188	248
Branch 2	47.62538	23.0911	55.32242	25.49642	83.46469	235
Branch 3	44.18014	21.42068	51.32037	23.652	77.42682	218
Branch 4	55.9345	27.11975	64.97441	29.94473	98.02661	276
Total	198	96	230	106	347	977

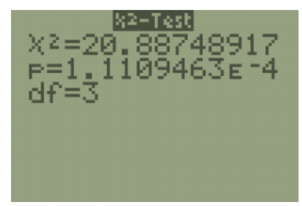


H_0: Type of electronic device and store branch are dependent.
H_1: Type of electronic device and store branch are dependent.
Test Statistic is $\chi^2 = \sum \frac{(O-E)^2}{E}$ = 7.4224;
p-value = 0.8285
Do not reject H_0.
There is not enough evidence to support the claim that type of electronic device and store branch are dependent.

33. A 4-year college is curious which of their students hold down a job while also attending school. They poll the students and find the results below. Test to see if there is a relationship between college students having a job and year in school. Use $\alpha = 0.05$.

Observed	**Freshmen**	**Sophomores**	**Juniors**	**Seniors**	**Total**
Works	61	45	31	72	209
Doesn't Work	45	33	38	19	135
Total	106	78	69	91	344

Observed	**Freshmen**	**Sophomores**	**Juniors**	**Seniors**	**Total**
Works	64.40116	47.38953	41.92151	55.28779	209
Doesn't Work	41.59884	30.61047	27.07849	35.71221	135
Total	106	78	69	91	344



H_0: Having a job in college and year in school are not related.
H_1: Having a job in college and year in school are related.
Test Statistic is $\chi^2 = \sum \frac{(O-E)^2}{E}$ = 20.8875;
p-value = 0.0001
Reject H_0.
There is enough evidence to support the claim that having a job in college and year in school are related.

Chapter 11 Exercises

1. What does the acronym ANOVA stand for? Answer a)
   a) Analysis of Variance
   b) Analysis of Means
   c) Analyzing Various Means
   d) Anticipatory Nausea and Vomiting
   e) Average Noise Variance

3. A researcher would like to test to see if there is a difference in the average profit between 5 different stores. Which are the correct hypotheses for an ANOVA? Answer e)

   a) H_0: $\mu_1 = \mu_2 = \mu_3$ — H_1: At least one mean is different.
   b) H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ — H_1: $\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4 \neq \mu_5$
   c) H_0: $\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4 \neq \mu_5$ — H_1: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$
   d) H_0: $\sigma_B^2 \neq \sigma_W^2$ — H_1: $\sigma_B^2 = \sigma_W^2$
   e) H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ — H_1: At least one mean is different.

5. An ANOVA was run for the per-pupil costs for private school tuition for three counties in the Portland, Oregon, metro area. Assume tuition costs are normally distributed. At $\alpha = 0.05$, test to see if there is a difference in the means.

SUMMARY				
Groups	*n*	*Sum*	*Average*	*Variance*
Clackamas County	11	147215	13383.1818	36734231.36
Multnomah County	12	182365	15197.0833	33731956.63
Washington County	10	124555	12455.5	40409869.17

   a) State the hypotheses.

   H_0: $\mu_1 = \mu_2 = \mu_3$
   H_1: At least one mean is different.

   b) Fill out the ANOVA table to find the test statistic.

   SSW = $\sum(n_i - 1)s_i^2$ = 10*36734231.36 + 11*33731956.63 + 9*40409869.17 = 1102082659.06

   $\bar{x}_{GM} = \frac{\sum x_i}{N}$ = (147215 + 182365 + 124555)/33 = 13761.6666667

   SSB = $\sum n_i(\bar{x}_i - \bar{x}_{GM})^2$ = 11(13383.1818 – 13761.6666667)^2 + 12(15197.0833 – 13761.6666667)^2 + 10(12455.5 – 13761.6666667)^2 = 43361523.2834

   dfW = k – 1 = 2; dfB = N – k = 33 – 3 = 30; dfTotal = N – 1 = 32
   MSB = SSB/dfB = 43361523.9/2 = 21680761.97
   MSW = SSW/dfW = 1102082659/30 = 36736088.64

   F = MSW/MSB = 21680761.97/36736088.64 = 0.590176112

ANOVA				
Source of Variation	*SS*	*df*	*MS*	*F*
Between Groups	43361523.2834	2	21680761.64	0.5901761
Within Groups	1102082659.06	30	36736088.64	
Total	1145444183.3434	32		

c) Find the p-value. =F.DIST.RT(0.5901761,2,30) = 0.5605

d) State the correct decision and summary. Do not reject H_0. There is not enough evidence to support the claim that there is a difference in the mean per-pupil costs for private school tuition for three counties in the Portland, Oregon, metro area.

7. What does the Bonferroni comparison test for? Answer c)
   a) The analysis of between and within variance.
   b) The difference between all the means at once.
   c) The difference between two pairs of mean.
   d) The sample size between the groups.

9. A manufacturing company wants to see if there is a significant difference in three types of plastic for a new product. They randomly sample prices for each of the three types of plastic and run an ANOVA. Use α = 0.05 to see if there is a statistically significant difference in the mean prices. Part of the computer output is shown below.

SUMMARY				
Groups	*Count*	*Sum*	*Average*	*Variance*
Plastic A	39	512	13.12821	15.48313
Plastic B	41	679	16.56098	1.302439
Plastic C	34	470	13.82353	22.08913

a) State the hypotheses.

H_0: $\mu_A = \mu_B = \mu_C$

H_1: At least one mean is different.

b) Fill in the ANOVA table to find the test statistic.

SSW = $\sum(n_i - 1)s_i^2$ = 38*15.48313 + 40*1.302439 + 33*22.08913 = 1369.39779

$\bar{x}_{GM} = \frac{\sum x_i}{N}$ = (512 + 679 + 470)/114 = 14.570175

SSB = $\sum n_i(\bar{x}_i - \bar{x}_{GM})^2$ = 39(13.12821 – 14.570175)^2 + 41(16.56098 – 14.570175)^2 + 34(13.82353 – 14.570175)^2 = 262.5410236

dfW = k – 1 = 2; dfB = N – k = 114 – 3 = 111; dfTotal = N – 1 = 113

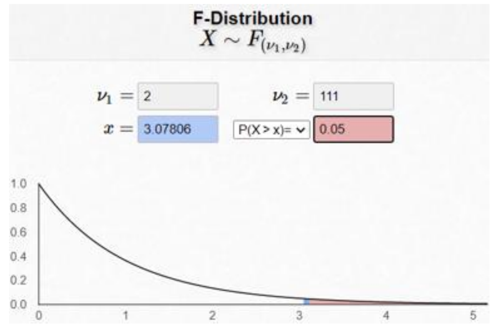
MSB = SSB/dfB = 262.5410236/2 = 131.2705118

MSW = SSW/dfW = 1369.39779/111 = 12.33691703

F = MSW/MSB = 131.2705118/12.33691703 = 10.64046

ANOVA				
Source of Variation	*SS*	*df*	*MS*	*F*
Between Groups	262.5410236	2	131.2705118	10.64046
Within Groups	1369.39779	111	12.33691703	
Total	1631.939	113		

c) Find the critical value. CV = F_α = 3.0781



d) State the decision and summary.

Reject H_0. There is enough evidence to support the claim that there is a difference in the mean price of the three types of plastic.

e) Which group(s) are significantly different based on the Bonferroni test?

Multiple Comparisons

Dependent Variable: Cost

Bonferroni

(I) Plastic Type	(J) Plastic Type	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Plastic A	Plastic B	-3.43277*	.78564	.000	-5.3425	-1.5230
	Plastic C	-.69532	.82412	1.000	-2.6986	1.3080
Plastic B	Plastic A	3.43277*	.78564	.000	1.5230	5.3425
	Plastic C	2.73745*	.81471	.003	.7570	4.7178
Plastic C	Plastic A	.69532	.82412	1.000	-1.3080	2.6986
	Plastic B	-2.73745*	.81471	.003	-4.7178	-.7570

H_0: $\mu_A = \mu_B$
H_1: $\mu_A \neq \mu_B$
p-value = 0; Reject H_0; There is significant difference in the mean price between plastics A and B.

H_0: $\mu_A = \mu_C$
H_1: $\mu_A \neq \mu_C$
p-value = 1; Do not reject H_0; There is not a significant difference in the mean price between plastics A and C.

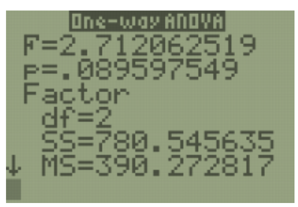
H_0: $\mu_B = \mu_C$
H_1: $\mu_B \neq \mu_C$
p-value = 0.003; Reject H_0; There is significant difference in the mean price between plastics B and C.

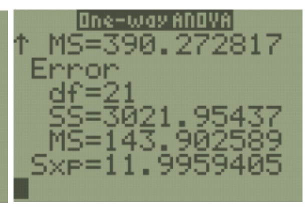
For exercises 11-15, Assume that all distributions are normal with equal population standard deviations, and the data was collected independently and randomly. Show all 5 steps for hypothesis testing. If there is a significant difference is found, run a Bonferroni test to see which means are different.

a) State the hypotheses.
b) Compute the test statistic.
c) Compute the critical value or p-value.
d) State the decision.
e) Write a summary.

11. Is a statistics class' delivery type a factor in how well students do on the final exam? The table below shows the average percent on final exams from several randomly selected classes that used the different delivery types. Assume that all distributions are normal with equal population standard deviations, and the data was collected independently and randomly. Use a level of significance of $\alpha = 0.10$.

Face-to-Face	Blended	Online
79	70	100
77	58	66
75	55	91
68	74	91
95	76	98
78	83	74
69	66	57
65		88
65		





$H_0: \mu_1 = \mu_2 = \mu_3$
H_1: At least one mean is different.

Source	SS	*df*	MS	F
Between	780.5456	2	390.2728	2.7121
Within	3021.9544	21	143.9026	
Total	3802.5	23		

$F = 2.7121$; p-value = 0.0896; Reject H_0.
There is sufficient evidence to support the claim that course delivery type is a factor in final exam score.

13. The dependent variable is movie ticket prices, and the groups are the geographical regions where the theaters are located (suburban, rural, urban). A random sample of ticket prices were taken from randomly chosen states. Test to see if there is a significant difference in the means using $\alpha = 0.05$.

Suburb	Rural	Urban
11.25	11.75	11.25
11	9.5	11.25
11	11.25	12.25
12.25	10.5	9.75
11.25	10	10.75
10	10	11.75
8.75	11.5	12
11	10.75	12.5
10.75	10.25	11
10.75	9.25	10.75
11.5	10.75	12
9.75	10	12
12.25	13	10.75
9.75	11	10.5
9.25	12	12.75

H_0: $\mu_1 = \mu_2 = \mu_3$
H_1: At least one mean is different.

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Suburb	15	160.5	10.7	1.0375
Rural	15	161.5	10.76667	1.022024
Urban	15	171.25	11.41667	0.71131

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	4.702778	2	2.351389	2.545865	0.090448	3.2199
Within Groups	38.79167	42	0.923611			
Total	43.49444	44				

F = 2.5459; p-value = 0.0904; Fail to reject H_0. There is not enough evidence to support the claim that there is a difference in the mean movie ticket prices by geographical regions where the theaters are located (suburban, rural, urban).

15. An ANOVA was run to test to see if there was a significant difference in the average cost between three different types of fabric for a new clothing company. Random samples for each of the three fabric types was collected from different manufacturers. Assume the costs are normally distributed. At $\alpha = 0.10$, run an ANOVA test to see if there is a difference in the

means. If a difference is found, run a Bonferroni test to see which means are different. Based off the Bonferroni results which fabric type should you choose?

SUMMARY

Groups	*n*	*Sum*	*Average*	*Variance*
A	34	10608	312	204.1212
B	37	11655	315	97.5
C	32	9600	300	2019.548

H_0: $\mu_A = \mu_B = \mu_C$
H_1: At least one mean is different.

ANOVA

Price

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	4217.417	2	2108.709	2.895	.060
Within Groups	72852.000	100	728.520		
Total	77069.417	102			

F = 2.895; p-value = 0.06; Reject H_0.
There is sufficient evidence to support the claim that there is a difference in the mean cost between three different types of fabric.

Multiple Comparisons

Dependent Variable: Price

Bonferroni

(I) Fabric Type	(J) Fabric Type	Mean Difference (I-J)	Std. Error	Sig.	90% Confidence Interval Lower Bound	90% Confidence Interval Upper Bound
Type A	Type B	-3.00000	6.41224	1.000	-16.8367	10.8367
	Type C	12.00000	6.64780	.222	-2.3450	26.3450
Type B	Type A	3.00000	6.41224	1.000	-10.8367	16.8367
	Type C	15.00000*	6.51583	.070	.9398	29.0602
Type C	Type A	-12.00000	6.64780	.222	-26.3450	2.3450
	Type B	-15.00000*	6.51583	.070	-29.0602	-.9398

H_0: $\mu_A = \mu_B$
H_1: $\mu_A \neq \mu_B$
p-value = 1; Do not reject H_0; There is not a significant difference in the mean cost of fabrics A and B.

H_0: $\mu_A = \mu_C$
H_1: $\mu_A \neq \mu_C$

p-value = 0.222; Do not reject H_0; There is not a significant difference in the mean cost of fabrics A and C.

H_0: $\mu_B = \mu_C$
H_1: $\mu_B \neq \mu_C$
p-value = 0.07; Reject H_0; There is significant difference in the mean cost of fabrics B and C.

17. For a two-way ANOVA, a row factor has 3 different levels, a column factor has 4 different levels. There are 15 data values in each group. Find the following.

a) The degrees of freedom for the row effect. $df_A = a - 1 = 2$, $dfE = 3*4*14 = 168$
b) The degrees of freedom for the column effect. $df_B = b - 1 = 3$, $dfE = 3*4*14 = 168$
c) The degrees of freedom for the interaction effect. $df_{A \times B} = (a - 1)(b - 1) = 2*3 = 6$; $dfE = 3*4*14 = 168$

19. Fill out the following two-way ANOVA table.

Source	Sum of Squares	d.f.	Mean Square	*F*
Row Factor	4357.17	2	2178.585	10.4072532
Column Factor	341.33	1	341.33	1.63055732
Interaction Factor	129.17	2	64.585	0.30852707
Error	1256	6	209.3333333	
Total	6083.67	11		

21. Fill out the following two-way ANOVA table.

Source	Sum of Squares	d.f.	Mean Square	*F*
Row Factor	10.08	1	10.08	2.41944194
Column Factor	720.75	1	720.75	172.9973
Interaction Factor	140.08	1	140.08	33.6225623
Error	33.33	8	4.16625	
Total	904.24	11		

23. A professor is curious if class size and format for which homework is administered has an impact on students' test grades. In a particular semester, she samples 4 students in each category below and records their grade on the department-wide final exam. The data are below recorded. Assume the variables are normally distributed. Run a two-way ANOVA using $\alpha = 0.05$.

	Less Than 30 Students	**More 30 Students**
Paper HW	92, 85, 72, 84	64, 72, 80, 88
Online HW	91, 90, 75, 78	82, 55, 62, 71

Source	Sum of Squares	d.f.	Mean Square	*F*
HW Factor	68.0625	1	68.0625	0.71849571
Class Size Factor	540.5625	1	540.5625	5.70639982
Interaction Factor	76.5625	1	76.5625	0.8082252
Error	1136.75	12	94.72916667	
Total	1821.9375	15		

H_0: The format of the homework (paper vs. online) has no effect on the mean test grade.
H_1: The format of the homework (paper vs. online) has an effect on the mean test grade.

F = 0.7185; CV = F.INV.RT(0.05,1,12) = 4.7472;
Do not reject H_0.
There is not enough evidence to support the claim that the format of the homework (paper vs. online) has an effect on the mean test grade.

H_0: The class size has no effect on the mean test grade.
H_1: The class size has an effect on the mean test grade.

F = 5.7064; CV = F.INV.RT(0.05,1,12) = 4.7472;
Reject H_0.
There is enough evidence to support the claim that the class size has an effect on the mean test grade.

H_0: There is no interaction effect between the format of the homework (paper vs. online) and the class size on the mean test grade.
H_1: There is an interaction effect between the format of the homework (paper vs. online) and the class size on the mean test grade.

F = 0.8082; CV = F.INV.RT(0.05,1,12) = 4.7472;
Do not reject H_0.
There is not enough evidence to support the claim that there is an interaction effect between the format of the homework (paper vs. online) and the class size on the mean test grade.

25. A door-to-door sales company sells three types of vacuums. The company manager is interested to find out if the type of vacuum sold has an effect on whether a sale is made, as well as what time of day the sale is made. She samples 36 sales representatives and divides them into the following categories, then records their sales (in hundreds of dollars) for a week. Assume the variables are normally distributed. Run a two-way ANOVA using α = 0.05.

	Vacuum 1	Vacuum 2	Vacuum 3
Morning	5.3, 4.2, 3.1, 4.8	6.2, 5.9, 7.1, 5.5	4.2, 3.9, 6.1, 4.8
Afternoon	4.8, 4.7, 5.1, 3.7	6.8, 7.2, 6.6, 5.3	4.1, 4.1, 5.3, 3.9
Evening	4.8, 4.9, 5.5, 5.7	7.5, 8.2, 9.1, 6.4	4.1, 5.2, 5.9, 4.3

Source	Sum of Squares	d.f.	Mean Square	*F*
Time of Day Factor	5.8472	2	2.9236	4.41792081
Vacuum Factor	36.2872	2	18.1436	27.4172212
Interaction Factor	2.3878	4	0.59695	0.9020652
Error	17.8675	27	0.661759259	
Total	62.3897	35		

H_0: The time of day has no effect on the mean number of vacuum sales.
H_1: The time of day has an effect on the mean number of vacuum sales.

F = 4.4179; CV = F.INV.RT(0.05,2,27) = 3.3541;
Reject H_0. There is enough evidence to support the claim that the time of day has an effect on the mean number of vacuum sales.

H_0: The type of vacuum has no effect on the mean number of vacuum sales.
H_1: The type of vacuum has an effect on the mean number of vacuum sales.

F = 27.4172; CV = F.INV.RT(0.05,2,27) = 3.3541;
Reject H_0. There is enough evidence to support the claim that the type of vacuum has an effect on the mean number of vacuum sales.

H_0: There is no interaction effect between time of day and type of vacuum on the mean number of vacuum sales.
H_1: There is an interaction effect between time of day and type of vacuum on the mean number of vacuum sales.

F = 0.9021; CV = F.INV.RT(0.05,4,27) = 2.7278;
Do not reject H_0. There is not enough evidence to support the claim that there is an interaction effect between time of day and type of vacuum on the mean number of vacuum sales.

27. A sample of patients are tested for cholesterol level and divided into categories by age and by location of residence in the United States. The data are recorded below. Assume the variables are normally distributed. A two-way ANOVA test was run and the information from the test is summarized in the table below. State all 3 hypotheses, critical values, decisions and summaries using $\alpha = 0.05$.

	East Coast	Midwest	South	West Coast
18-35 yrs.	185, 193, 172	202, 193, 187	220, 218, 198	184, 199, 203
36-53 yrs.	202, 218, 199	205, 219, 215	205, 224, 229	205, 187, 190
54+ yrs.	222, 231, 206	215, 209, 197	225, 233, 214	184, 212, 216

Source	Sum of Squares	d.f.	Mean Square	*F*
Age Factor	1922.000	2	961.000	7.863
Location Factor	2093.333	3	697.778	5.709
Interaction Factor	1067.333	6	177.889	1.455
Error	2933.333	24	122.222	
Total	8016.000	35		

H_0: Age has no effect on the mean cholesterol level.
H_1: Age has an effect on the mean cholesterol level.

F = 7.863; CV = F.INV.RT(0.05,2,24) = 3.4028; Reject H_0. There is enough evidence to support the claim that age has an effect on the mean cholesterol level.

H_0: Location has no effect on the mean cholesterol level.
H_1: Location has no effect on the mean cholesterol level.

F = 5.709; CV = F.INV.RT(0.05,3,24) = 3.0087; Reject H_0. There is enough evidence to support the claim that the location has an effect on the mean cholesterol level.

H_0: There is no interaction effect between age and location on the mean cholesterol level.
H_1: There is an interaction effect between age and location on the mean cholesterol level.

F = 1.455; CV = F.INV.RT(0.05,6,24) = 2.5082; Do not reject H_0. There is not enough evidence to support the claim that there is an interaction effect between age and location on the mean cholesterol level.

29. An obstetrician feels that her patients who are taller and leaner before becoming pregnant typically have quicker deliveries. She samples 3 women in each of the following categories of Height and Body Mass Index and records the time they spent in the pushing phase of labor in minutes. All women in the sample had a natural vaginal delivery and it was their first childbirth. The data are recorded below. Assume the variables are normally distributed. Run a two-way ANOVA using $\alpha = 0.05$.

	BMI 20-24	**BMI 25-29**	**BMI 30-35**
58 – 63 in.	143, 95, 162	212, 51, 146	208, 162, 84
64 – 69 in.	165, 45, 130	133, 42, 110	125, 137, 162
70 – 75 in.	89, 102, 35	46, 33, 114	95, 125, 110

Source	Sum of Squares	d.f.	Mean Square	*F*
Height Factor	14814.5185	2	7407.25925	3.27978884
BMI Factor	6216.5185	2	3108.25925	1.37627612
Interaction Factor	1016.5926	4	254.14815	0.11253181
Error	42910.6667	19	2258.456142	
Total	64958.2963	27		

H_0: Height has no effect on the mean delivery time.
H_1: Height has an effect on the mean delivery time.

F = 3.2798; CV = F.INV.RT(0.05,2,19) = 3.5219; Do not reject H_0. There is not enough evidence to support the claim that height has an effect on the mean delivery time.

H_0: BMI has no effect on the mean delivery time.
H_1: BMI has an effect on the mean delivery time.

F = 1.3763; CV = F.INV.RT(0.05,2,19) = 3.5219; Do not reject H_0. There is not enough evidence to support the claim that BMI has an effect on the mean delivery time.

H_0: There is no interaction effect between the height and BMI on the mean delivery time.
H_1: There is an interaction effect between the height and BMI on the mean delivery time.

F = 0.1125; CV = F.INV.RT(0.05,4,19) = 2.8951; Do not reject H_0. There is not enough evidence to support the claim that there is an interaction effect between the height and BMI on the mean delivery time.

Chapter 12 Exercises

1. The correlation coefficient, r, is a number between __________. Answer: a)
   a) -1 and 1
   b) -10 and 10
   c) 0 and 10
   d) 0 and ∞
   e) 0 and 1
   f) -∞ and ∞

3. What are the hypotheses for testing to see if a correlation is statistically significant?
   a) H_0: $r = 0$ H_1: $r \neq 0$
   b) H_0: $\rho = 0$ H_1: $\rho \neq 0$
   c) H_0: $\rho = \pm 1$ H_1: $\rho \neq \pm 1$
   d) H_0: $r = \pm 1$ H_1: $r \neq \pm 1$
   e) H_0: $\rho = 0$ H_1: $\rho = 1$

   Answer: b)

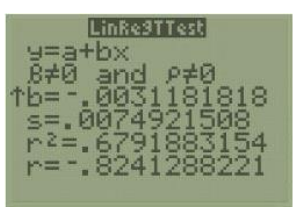
5. Which of the following is not a valid linear regression equation?
   a) $\hat{y} = -5 + \frac{2}{9}x$
   b) $\hat{y} = 3x + 2$
   c) $\hat{y} = \frac{2}{9} - 5x$
   d) $y = 5 + 0.4x$

   Answer: d)

   A regression equation needs the hat over the y for the predicted value $\hat{y}$.

7. Bone mineral density and cola consumption has been recorded for a sample of patients. Let x represent the number of colas consumed per week and y the bone mineral density in grams per cubic centimeter. Assume the data is normally distributed. Calculate the correlation coefficient. $r = 0.8241$

x	y
1	0.883
2	0.8734
3	0.8898
4	0.8852
5	0.8816
6	0.863
7	0.8634
8	0.8648
9	0.8552
10	0.8546
11	0.862



9. The sum of the residuals should be __________.
   a) a
   b) b
   c) 0
   d) 1
   e) r

   Answer: c)
   Residuals are positive when *above* the regression line and negative when *below* the regression line. The regression line is designed so that the sum of the residuals will all cancel out and the result will be 0.

11. A teacher believes that the third homework assignment is a key predictor in how well students will do on the midterm. Let x represent the third homework score and y the midterm exam score. A random sample of last terms students were selected and their grades are shown below. Assume scores are normally distributed.

HW3	Midterm	HW3	Midterm	HW3	Midterm
13.1	59	6.4	43	20	86
21.9	87	20.2	79	15.4	73
8.8	53	21.8	84	25	93
24.3	95	23.1	92	9.7	52
5.4	39	22	87	15.1	70
13.2	66	11.4	54	15	65
20.9	89	14.9	71	16.8	77
18.5	78	18.4	76	20.1	78

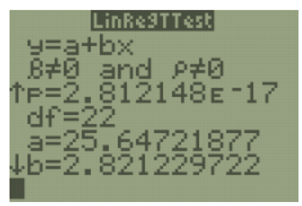
a) Compute the regression equation.

Compute the 2-Var Stats and sum of squares.

$SS_{xx} = (n-1)s_x^2 = (24-1)5.558814322^2 = 710.7095833$

$SS_{yy} = (n-1)s_y^2 = (24-1)15.97892634^2 = 5872.5$

$SS_{xy} = \sum(xy) - n \cdot \bar{x} \cdot \bar{y} = 31156 - 24 \cdot 16.695833 \cdot 72.75 = 2005.075$



Calculate the slope: $b_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{2005.075}{710.7095833} = 2.821229722.$

Then calculate the y-intercept: $b_0 = \bar{y} - b_1 \cdot \bar{x} = 72.75 - (2.821229722) \cdot 16.695833 = 25.64721877.$

Put the numbers back into the regression equation.

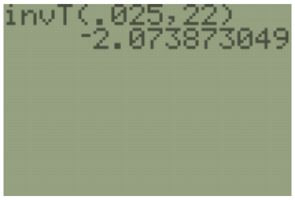
Write your answer as: $\hat{y} = 25.6472 + 2.8212x$

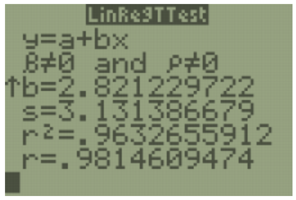
b) Compute the predicted midterm score when the homework 3 score is 15.

$\hat{y} = 25.6472 + 2.8212 * 15 = 67.96566$

c) Compute the residual for the point (15, 65). $y - \hat{y} = 65 - 67.96566 = -2.96566$

d) Find the 95% prediction interval for the midterm score when the homework 3 score is 15.





$\hat{y} \pm t_{\alpha/2} \cdot s \cdot \sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{SS_{xx}}}$ $\quad 67.96566 \pm 2.073873 \cdot 3.1313867 \cdot \sqrt{1 + \frac{1}{24} + \frac{(15-16.695833)^2}{710.7095833}}$

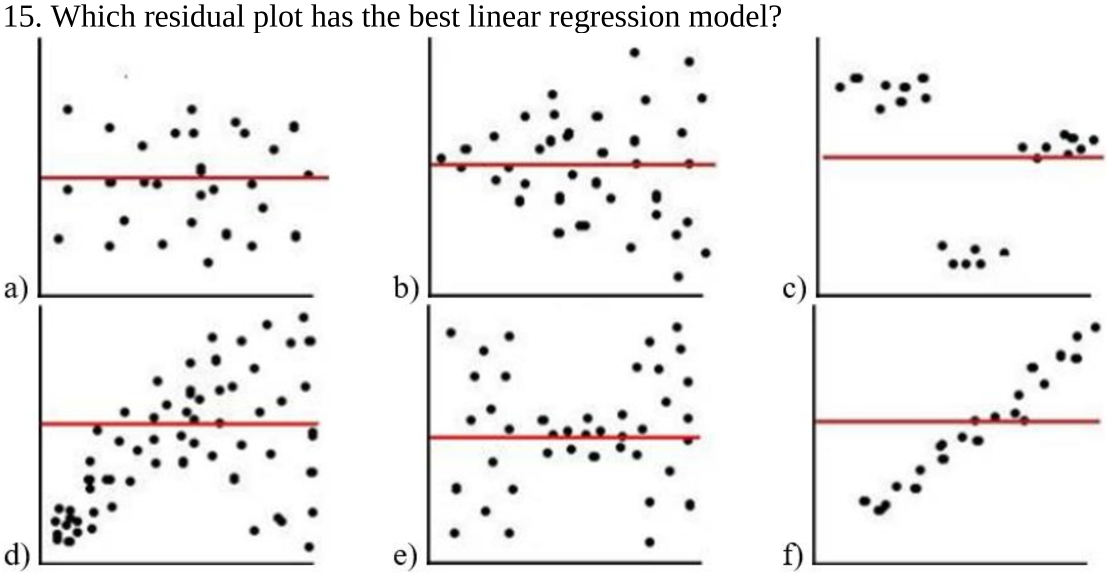
67.96566 ± 6.640873

61.3248 < y < 74.6065

13. Bone mineral density and cola consumption have been recorded for a sample of patients. Let x represent the number of colas consumed per week and y the bone mineral density in grams per cubic centimeter. Assume the data is normally distributed. Calculate the coefficient of determination. $r^2 = R^2 = 0.679$

x	**y**
1	0.883
2	0.8734
3	0.8898
4	0.8852
5	0.8816
6	0.863
7	0.8634
8	0.8648
9	0.8552
10	0.8546
11	0.862

15. Which residual plot has the best linear regression model?

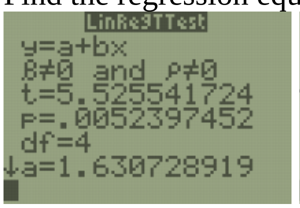


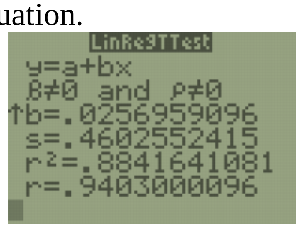
a) a
b) b
c) c
d) d
e) e
f) f
Answer: a)
A residual plot should have no pattern and the data points on the graph should vary in distance from the red line in the middle of the graph for it to represent a linear model.

17. The following data represent the leaching rates (percent of lead extracted vs. time in minutes) for lead in solutions of magnesium chloride ($MgCl_2$).

Time (x)	4	8	16	30	60	120
Percent Extracted (y)	1.2	1.6	2.3	2.8	3.6	4.4

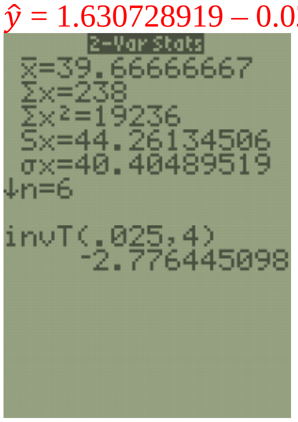
a) State the hypotheses to test for a significant correlation. $H_0: \rho = 0$; $H_1: \rho \neq 0$
b) Find the correlation coefficient. $r = 0.9403$
c) Find the p-value to see if there is a significant correlation. p = 0.0052
d) State the correct decision. Reject H_0
e) Is there a significant correlation? Yes
f) Find the coefficient of determination. $R^2 = 0.8842$
g) Find the regression equation.

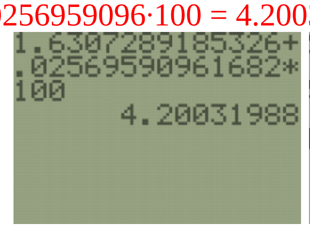


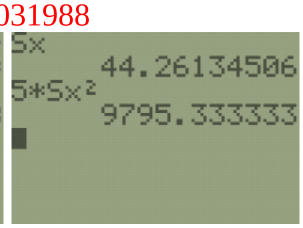


$\hat{y} = 1.630728919 – 0.0256959096x$

h) Find the 95% prediction interval for 100 minutes.
$\hat{y} = 1.630728919 – 0.0256959096 \cdot 100 = 4.20031988$







$SS_{xx} = (n – 1)*s^2 = 5*4426134506^2 = 9795.3333333$
s = 0.4602552415

$$\hat{y} \pm t_{\alpha/2} \cdot s \cdot \sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{SS_{xx}}}$$

$$4.20031988 \pm 2.776445098 \cdot 0.4602552415 \cdot \sqrt{1 + \frac{1}{6} + \frac{(100-39.66666667)^2}{9795.3333333}}$$

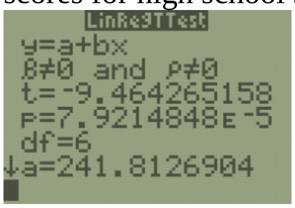
4.20031988 ± 1.584915
$2.6154 < y < 5.7852$

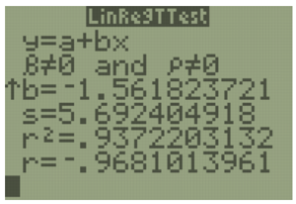
i) Write a sentence interpreting this interval using units and context. We can be 95% confident that the predicted percent of lead extracted in solutions of magnesium chloride at 100 minutes is anywhere between 2.6154 and 5.7852.

19. A study was conducted to determine if there was a linear relationship between a person's age and their peak heart rate. Use α=0.05.

Age (x)	16	26	32	37	42	53	48	21
Peak Heart Rate (y)	220	194	193	178	172	160	174	214

a) What is the estimated regression equation that relates number of hours worked and test scores for high school students.





$\hat{y}$ = 241.8127 – 1.5618x

b) Interpret the slope coefficient for this problem. Every year a person ages, their peak heart rate decreases by an average of 1.5618.

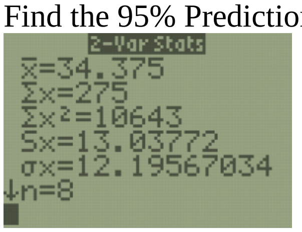
c) Compute and interpret the coefficient of determination. R^2 = 0.93722

d) Compute the coefficient of nondetermination. 1 – R^2 = 1 – 0.93722 = 0.06278

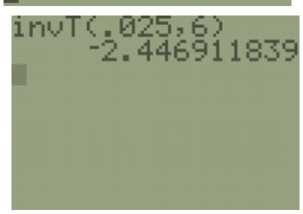
e) Compute the standard error of estimate. s = 5.692404918

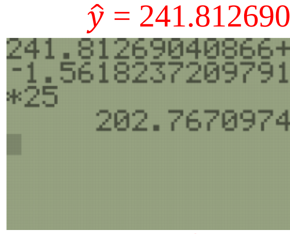
f) Compute the correlation coefficient. r = -0.9681

g) Find the 95% Prediction Interval for peak heart rate for someone who is 25 years old.



$\hat{y}$ = 241.8126904 – 1.561823721·25 = 202.7670974





$t_{\alpha/2}$ = -2.446911839, SS_{xx} = (n – 1)*s^2 = 7*13.03772^2 = 1189.875

s = 5.692404918

$$\hat{y} \pm t_{\alpha/2} \cdot s \cdot \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{SS_{xx}}}$$

$$202.7670974 \pm 2.446911839 \cdot 5.692404918 \cdot \sqrt{1 + \frac{1}{8} + \frac{(25 - 34.375)^2}{1189.875}}$$

202.7670974 ± 15.25103527

187.5161 < y < 218.0181

21. Body frame size is determined by a person's wrist circumference in relation to height. A researcher measures the wrist circumference and height of a random sample of individuals. The Excel output and scatterplot are displayed below. Find the regression equation and predict the height (in inches) for a person with a wrist circumference of 7 inches. Then, compute the residual for the point (7, 75).

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	31.6304	5.2538	6.0205	1.16E-06
Circumference	5.4496	0.7499	7.2673	3.55E-08

$\hat{y} = 31.6304 + 5.4496x$ = 69.7776 when x = 7.
The predicted height (in inches) for a person with a wrist circumference of 7 inches is 69.7776. The residual is $y - \hat{y} = 75 - 69.7776 = 5.2224$

23. The data below represent the driving speed (mph) of a vehicle and the corresponding gas mileage (mpg) for several recorded instances.

Driving Speed	**Gas Mileage**
57	21.8
66	20.9
42	25.0
34	26.2
44	24.3
44	26.3
25	26.1
20	27.2
24	23.5
42	22.6
52	19.4
54	23.9
60	24.8
62	21.5
66	20.5
67	23.0
52	24.2
49	25.3
48	24.3
41	28.4
38	29.6
26	32.5
24	30.8
21	28.8
19	33.5
24	25.1

a) Do a hypothesis test to see if there is a significant correlation. Use α = 0.10.

H_0: $\rho = 0$; H_1: $\rho \neq 0$

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	32.40313	1.426646	22.7128	9.81E-18
Driving Speed	-0.1662	0.031648	-5.25143	2.2E-05

p-value = 0.000022, which is less than α = 0.10, so reject H_0. There is a significant correlation between driving speed of a vehicle and the corresponding gas mileage.

b) Compute the standard error of estimate.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.731219
R Square	0.534681
Adjusted R Square	0.515293
Standard Error	2.49433
Observations	26

$s = 2.49433$

c) Compute the regression equation and use it to find the predicted gas mileage when a vehicle is driving at 77 mph.

	Coefficients
Intercept	32.40313
Driving Speed	-0.1662

$\hat{y} = 32.40313 - 0.1662x$

d) Compute the 90% prediction interval for gas mileage when a vehicle is driving at 77 mph.

$\hat{y} = 32.40313 - 0.1662 \cdot 77 = 19.60606$ mpg

Use $\hat{y} \pm t_{\alpha/2} \cdot s \cdot \sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{SS_{xx}}}$

$t_{\alpha/2}$ = T.INV(0.05,24) = –1.71088

$SS_{xx} = (n-1) \cdot s_x^2 = (26-1) \cdot 248.4753846 = 6211.884615$

Driving Speed	
Mean	42.34615385
Standard Error	3.091398642
Median	43
Mode	24
Standard Deviation	15.763102
Sample Variance	248.4753846

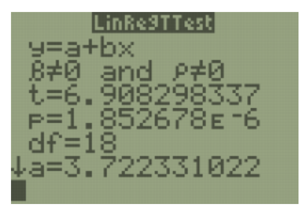
$19.60606 \pm 1.71088 \cdot 2.49433 \cdot \sqrt{1+\frac{1}{26}+\frac{(77-42.34615385)^2}{6211.884615}}$

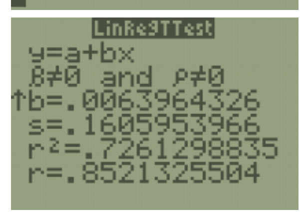
Answer: $14.8697 < y < 24.3424$

25. In a sample of 20 football players for a college team, their weight and 40-yard-dash time in minutes was recorded.

Weight (lbs.)	40-Yard-Dash	Weight (lbs.)	40-Yard-Dash
285	5.95	195	4.85
185	4.99	254	5.12
165	4.92	140	4.87
188	4.77	212	5.05
160	4.52	158	4.75
156	4.67	188	4.87
256	5.22	134	4.53
169	4.95	205	4.92
210	5.06	178	4.88
165	4.83	159	4.79

a) Do a hypothesis test to see if there is a significant correlation. Use α = 0.01.
H_0: ρ = 0; H_1: ρ ≠ 0
p-value = 0.00000185
Reject H_0
There is a significant correlation between a football player's weight and 40-yard-dash time.

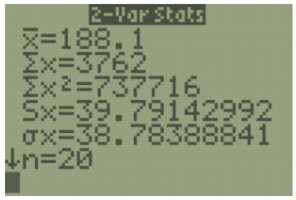


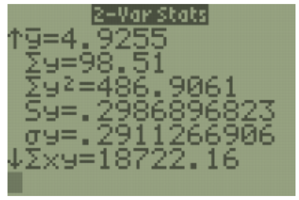


b) Compute the standard error of estimate.
$s = 0.160595$

c) Compute the regression equation and use it to find the predicted 40-yard-dash time for a football player that is 200 lbs.
$\hat{y} = 3.722331022 + 0.0063964326 \cdot 200 = 5.001617542$

d) Compute the 99% prediction interval for a football player that is 200 lbs.
Use $\hat{y} \pm t_{\alpha/2} \cdot s \cdot \sqrt{1+\frac{1}{n}+\frac{(x-\bar{x})^2}{SS_{xx}}}$





$SS_{xx} = (n-1) \cdot s_x^2 = (20-1) \cdot 39.79142992^2 = 30083.80001$

$5.001617542 \pm 2.878 \cdot 0.160595 \cdot \sqrt{1 + \frac{1}{20} + \frac{(200-188.1)^2}{30083.80001}}$

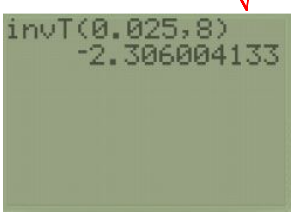
Answer: $4.527 < y < 5.476$

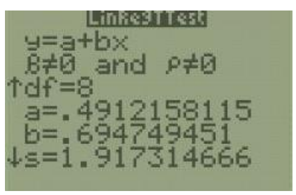
e) Write a sentence interpreting the prediction interval.
We can be 99% confident that the predicted time for all 200-pound football player's running time for the 40-yard-dash is between 4.527 and 5.476 minutes.

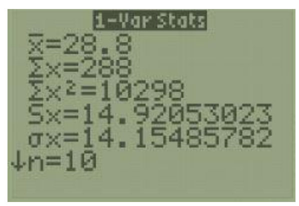
27. A new fad diet called Trim-to-the-MAX is running some tests that they can use in advertisements. They sample 25 of their users and record the number of days each has been on the diet along with how much weight they have lost in pounds. The data is below. A significant linear correlation was found between the two variables. Find the 95% prediction interval for the weight lost when a person has been on the diet for 60 days.

Days on Diet	7	12	16	19	25	34	39	43	44	49
Weight Lost	5	7	12	15	20	25	24	29	33	35

Use $\hat{y} \pm t_{\alpha/2} \cdot s \cdot \sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{SS_{xx}}}$







$t_{\alpha/2}$ = -2.306, $SS_{xx} = (n-1) \cdot s_x^2 = (10-1) \cdot 14.92053023^2 = 2003.600001$

$\hat{y} = 0.4912158115 + 0.694749451 \cdot 60 = 42.17618287$

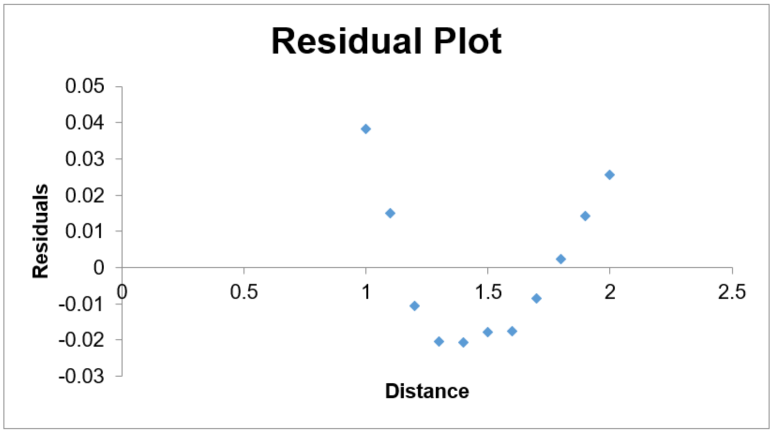
$42.17618287 \pm 2.306 \cdot 1.917315 \cdot \sqrt{1 + \frac{1}{10} + \frac{(60-28.8)^2}{2003.600001}}$

Answer: $36.605 < y < 47.740$

29. The intensity (in candelas) of a 100-watt light bulb was measured by a sensing device at various distances (in meters) from the light source. A linear regression was run and the following residual plot was found.

Regression Statistics	
Multiple R	0.95936
R Square	0.920371
Adjusted R Square	0.911523
Standard Error	0.021636
Observations	11

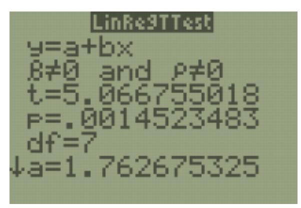
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.468618	0.031624	14.81856	1.25E-07
Distance	-0.2104	0.020629	-10.1992	3.04E-06

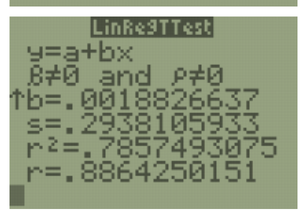


a) Is linear regression a good model to use? No, since there is a U shape in the residual plot.
b) Write a sentence explaining your answer. The p-value = 0.00000304 suggests that there is a significant linear relationship between intensity (in candelas) of a 100-watt light bulb was measured by a sensing device at various distances (in meters) from the light source. However, the residual plot clearly shows a nonlinear relationship. Even though we can fit a straight line through the points, we would get a better fit with a curve.

31. A nutritionist feels that what mothers eat during the months they are nursing their babies is important for healthy weight gain of their babies. She samples several of her clients and records their average daily caloric intake for the first three months of their babies' lives and also records the amount of weight the babies gained in those three months. The data are below.

Daily Calories	1523	1649	1677	1780	1852	2065	2096	2145	2378
Baby's Weight Gain (lbs.)	4.62	4.77	4.62	5.12	5.81	5.34	5.89	5.96	6.05





a) Compute the regression equation.
$\hat{y} = 1.762675325 + 0.0018826637 \cdot x$

b) Test to see if the slope is significantly different from zero, use α = 0.05.
H_0: $\beta_1 = 0$; H_1: $\beta_1 \neq 0$
p-value = 0.00145
Reject H_0
There is significant linear relationship between a nursing baby's weight gain and the calorie intake of the mother.

c) Predict the weight gain of a baby whose mother gets 2,500 calories per day.
$\hat{y} = 1.762675325 + 0.0018826637 \cdot 2500 = 6.469334575$

d) Compute the 95% prediction interval for the weight gain of a baby whose mother gets 2,500 calories per day.

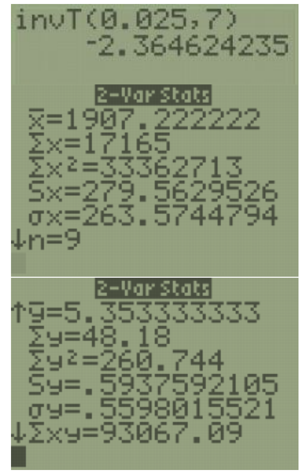
Use $\hat{y} \pm t_{\alpha/2} \cdot s \cdot \sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{SS_{xx}}}$

$t_{\alpha/2}$ = -2.365,

$SS_{xx} = (n-1) \cdot s_x^2 = (9-1) \cdot 279.5629526^2 = 625243.5557$

$6.469334575 \pm 2.365 \cdot 0.293811 \cdot \sqrt{1 + \frac{1}{9} + \frac{(2500-1907.222222)^2}{625243.5557}}$

Answer: 5.571 < y < 7.368



33. In a multiple linear regression problem, p represents:
a) The number of dependent variables in the problem.
b) The probability of success.
c) The number of independent variables in the problem.
d) The probability of failure.
e) The population proportion.

Answer: c)

35. A multiple regression test concludes that there is a linear relationship and finds the following line of best fit: $\hat{y}$ = –53.247 + 12.594x_1 – 0.648x_2 + 4.677x_3. Use the line of best fit to approximate y when x_1 = 5, x_2 = 12, x_3 = 2.

$\hat{y}$ = –53.247 + 12.594x_1 – 0.648x_2 + 4.677x_3 = –53.247 + 12.594·5 – 0.648·12 + 4.677·2 = 11.301

37. A study conducted by the American Heart Association provided data on how age, blood pressure and smoking relate to the risk of strokes. The following data is the SPSS output with Age (in years x_1), Blood Pressure (mmHg x_2), Smoker (0=Nonsmoker, 1=Smoker x_3) and the Risk of a Stroke as a percent (y).

SUMMARY OUTPUT

Regression Statistics	
Multiple R	
R Square	0.873
Adjusted R Square	
Standard Error	5.75657
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	3660.740	120.247	36.823	2.04E-07
Residual	16	530.210	33.138		
Total	19	4190.950			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-91.759	15.223	-6.028	1.7568E-05
Age	1.077	0.166	6.488	7.4848E-07
Blood Pressure	0.252	0.045	5.568	4.2433E-05
Smoker	8.740	3.001	2.912	0.0102

a) Use α = 0.05 to test the claim that the regression model is significant. State the hypotheses, test statistic, p-value, decision and summary.
H_0: $\beta_1 = \beta_2 = \beta_3 = 0$; H_1: At least one slope is not zero.
Test statistic = F = 36.823
p-value = 0.000000204
Reject H_0
There is a significant relationship between a person's age, blood pressure and smoking status and the risk of a stroke.

b) Use the estimated regression equation to predict the stroke risk for a 70-year-old smoker with a blood pressure of 180.
$\hat{y} = -91.759 + 1.077x_1 + 0.252x_2 + 8.74x_3 = -91.759 + 1.077 \cdot 70 + 0.252 \cdot 180 + 8.74 \cdot 1 =$ 37.731%

c) Interpret the slope coefficient for age.
Holding all other variables constant, for each year a person ages, their chance of a stroke increases by 1.077%.

d) Find the adjusted coefficient of determination.
$R^2_{adj} = 1 - \left(\frac{(1-R^2)(n-1)}{(n-p-1)}\right) = 1 - \left(\frac{(1-0.873)(19)}{(16)}\right)$ = 0.849 or 84.9%

39. A study was conducted to determine if there was a linear relationship between a person's weight in pounds with their gender, height and activity level. A person's gender was recorded as a 0 for anyone who identified as male and 1 for those who did not identify as male, height was in inches, activity level was coded as 1, 2, or 3, the more active the higher the value.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	
R Square	0.667
Adjusted R Square	
Standard Error	13.9353
Observations	92

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	34195	11398.33	58.6958	6.2026E-21
Residual	88	17089	194.1932		
Total	91	51284			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-81.5	43.96	-1.854	0.0671
Gender	-15.1	4.286	-3.5231	0.0007
Height	3.7	0.5704	6.4867	4.97E-09
Activity Level	-3.08	2.443	-1.2607	0.2107

a) Interpret the slope coefficient for height.
Holding all other variables constant, for each additional inch in height, the predicted weight would increase 3.7 pounds.

b) Predict the weight for a male who is 70 inches tall and has an activity level of 2?
$\hat{y} = -81.5 - 15.1x_1 + 3.7x_2 - 3.08x_3 = -81.5 - 15.1\cdot 0 + 3.7\cdot 70 - 3.08\cdot 2 = 171.34$ pounds

c) Calculate the adjusted coefficient of determination?
$R^2_{adj} = 1 - \left(\frac{(1-R^2)(n-1)}{(n-p-1)}\right) = 1 - \left(\frac{(1-0.667)(91)}{(88)}\right)$ = 0.6556 or 65.56%

Chapter 13 Exercises

For exercises 1-5, show all 5 steps for hypothesis testing:

a) State the hypotheses.
b) Compute the test statistic.
c) Compute the critical value or p-value.
d) State the decision.
e) Write a summary.

1. A real estate agent suggests that the median rent for a one-bedroom apartment in Portland has changed from last year's median of \$825 per month. A sample of 12 one-bedroom apartments shows these monthly rents in dollars for a one-bedroom apartment; 820, 720, 960, 660, 735, 910, 825, 1050, 915, 905, 1050, 950. Is there enough evidence to claim that the median rent has changed from \$825? Use $\alpha = 0.05$.

   H_0: Median = 825
   H_1: Median $\neq$ 825

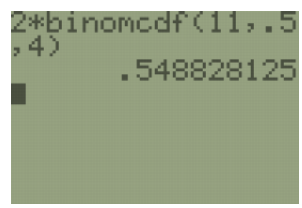
820	720	960	660	735	910	825	1050	915	905	1050	950
–	–	+	–	–	+	0	+	+	+	+	+

   There are 4 – signs, 7 + signs, and 1 tie, so n = 11.
   Test Statistic = 4

   $P(X \leq 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) =$
   ${}_{11}C_0 \cdot 0.5^0 \cdot 0.5^{11} + {}_{11}C_1 \cdot 0.5^1 \cdot 0.5^{10} + {}_{11}C_2 \cdot 0.5^2 \cdot 0.5^9 + {}_{11}C_3 \cdot 0.5^3 \cdot 0.5^8 + {}_{11}C_4 \cdot 0.5^4 \cdot 0.5^7 = 0.000488 + 0.00537 + 0.02686 + 0.08057 + 0.16113 = 0.2744$.

   Since this is a two-tailed test we multiply the probability by 2 to get 2*0.2744 = 0.5488.

   Using the TI-84 calculator for a two-tailed test we get 2*binomcdf(11,0.5,4) = 0.5548.



   The p-value = 0.5548

   Do not reject H_0.

   There is not enough evidence to support the claim that the median rent has changed from last year's median of \$825 per month.

3. A meteorologist believes that the median temperature for the month of July in Jacksonville, Florida, is higher than the previous year of 81˚F. The following sample shows the temperatures taken at noon in Jacksonville during July. Is there enough evidence to support the meteorologist's claim? Use $\alpha = 0.05$.

79	85	81	95	80	98	82	81	76	84	90	93
–	+	tie	+	–	+	+	tie	–	+	+	+

H_0: Median = 81
H_1: Median > 81

There are 3 – signs, 7 + signs, and 2 ties, so n = 10.
Test Statistic = 7

$P(X \geq 7) = 1 - \text{binomcdf}(10,0.5,7) = 0.0547$.
The p-value = 0.0547

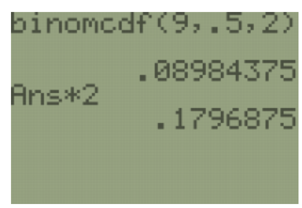
Do not reject H_0.

There is not enough evidence to support the claim that the median temperature for the month of July in Jacksonville, Florida, is higher than the previous year of 81˚F.

5. Test to see if the median assessed property value ($1,000) changed between 2010 and 2016. Use the sign test and $\alpha = 0.05$.

Ward	A	B	C	D	E	F	G	H	I	J	K
2010	184	414	22	99	116	49	24	50	282	25	141
2016	161	382	22	190	120	52	28	50	297	40	148
Difference Sign	+	+	0	–	–	–	–	0	–	–	–

H_0: There is no change in the median assessed property value between 2010 and 2016.
H_1: There is a change in the median assessed property value between 2010 and 2016.



There are 7 – signs, 2 + signs, and 2 ties, so n = 9.
Test Statistic = 2

$P(X \leq 3) = \text{binomcdf}(9,0.5,2) = 0.1797$.
The p-value = 0.1797.

Do not reject H_0.

There is not enough evidence to support the claim that is a change in the median assessed property value between 2010 and 2016.

7. Rank the following data set: -15, -25, -5, -5, -8, -15, -2, -4, -5.

Ordered Data	Rank
-25	1
-15	2.5
-15	2.5
-8	4
-5	6
-5	6

-5	6
-4	8
-2	9

9. Rank the following data set: 1, 2, 9, 3, 5, 1, 2, 8, 6.

Ordered Data	Rank
1	1.5
1	1.5
2	3.5
2	3.5
3	5
5	6
6	7
8	8
9	9

For exercises 11-19, show all 5 steps for hypothesis testing:

a) State the hypotheses.
b) Compute the test statistic.
c) Compute the critical value or p-value.
d) State the decision.
e) Write a summary.

11. A manager wishes to see if the time (in minutes) it takes for their workers to complete a certain task will decrease when they are allowed to wear earbuds at work. A random sample of 20 workers' times were collected before and after. Test the claim that the time to complete the task has decreased at a significance level of $\alpha = 0.01$ using the Wilcoxon Signed-Rank test. For the context of this problem, the first data set represents before measurement and the second data set represents the after measurement. You obtain the following sample data.

H_0: The time to complete the task will not decrease when workers are allowed to wear earbuds.
H_1: The time to complete the task will decrease when workers are allowed to wear earbuds.

This is a right tailed test, since we want Before time > After time.

Before	After	D	|D|	Rank	Signed Rank
69	62.3	6.7	6.7	6	6
71.5	61.6	9.9	9.9	13	13
39.3	21.4	17.9	17.9	19	19
67.7	60.4	7.3	7.3	8	8
38.3	47.9	-9.6	9.6	12	-12
85.9	77.6	8.3	8.3	11	11

67.3	75.1	-7.8	7.8	10	-10
59.8	46.3	13.5	13.5	18	18
72.1	65	7.1	7.1	7	7
79	83	-4	4	2	-2
61.7	56.8	4.9	4.9	3.5	3.5
55.9	44.7	11.2	11.2	14	14
56.8	50.6	6.2	6.2	5	5
71	63.4	7.6	7.6	9	9
80.6	68.9	11.7	11.7	16.5	16.5
59.8	35.5	24.3	24.3	20	20
72.1	77	-4.9	4.9	3.5	-3.5
49.9	38.4	11.5	11.5	15	15
56.2	55.4	0.8	0.8	1	1
63.3	51.6	11.7	11.7	16.5	16.5

The test statistic is $w_s = 27.5$.

In Figure 13-5, use the one-tail column for $\alpha = 0.01$, $n = 20$ row, to correspond with critical value of 43.

Since the test statistic is less than the critical value, reject H_0.

There is enough evidence that allowing the workers to wear earbuds significantly decreased the time for workers to complete tasks.

13. An adviser is testing out a new online learning module for a placement test. They wish to test the claim that the new online learning module increased placement scores at a significance level of $\alpha = 0.05$. You obtain the following paired sample of 19 students who took the placement test before and after the learning module. Use the Wilcoxon Signed-Rank test.

Before	After	D	|D|	Rank	Signed Rank
55.8	57.1	-1.3	1.3	1	-1
51.7	58.3	-6.6	6.6	10	-10
76.6	83.6	-7	7	12	-12
47.5	49.5	-2	2	4	-4
48.6	51.1	-2.5	2.5	5	-5
11.4	20.6	-9.2	9.2	15	-15
30.6	35.2	-4.6	4.6	8	-8
53	46.7	6.3	6.3	9	9
21	22.5	-1.5	1.5	2	-2
58.5	47.7	10.8	10.8	16	16
42.6	51.5	-8.9	8.9	13.5	-13.5
61.2	76.6	-15.4	15.4	19	-19
26.8	28.6	-1.8	1.8	3	-3

11.4	14.5	-3.1	3.1	6	-6
56.3	43.7	12.6	12.6	18	18
46.1	57	-10.9	10.9	17	-17
72.8	66.1	6.7	6.7	11	11
42.2	38.1	4.1	4.1	7	7
51.3	42.4	8.9	8.9	13.5	13.5

H_0: The new online learning module did not increase student's placement scores.
H_1: The new online learning module increased student's placement scores.

The test statistic $w_s = 74.5$.

This is a left-tailed test since we want Before < After. Use the $\alpha = 0.05$ column, n = 19 row in Figure 13-5 to get a critical value of 53.

Do not reject H_0.

There is not enough evidence to support the claim that the new online learning module increased student's placement scores.

15. In Major League Baseball, the American League (AL) allows a designated hitter (DH) to bat in place of the pitcher, but in the National League (NL), the pitcher has to bat. However, when an AL team is the visiting team for a game against an NL team, the AL team must abide by the home team's rules and thus, the pitcher must bat. A researcher is curious if an AL team would score differently for games in which the DH was used. She samples 20 games for an AL team for which the DH was used, and 20 games for which there was no DH. The data are below. Use the Mann-Whitney test with $\alpha = 0.05$.

With DH	Rank	Without DH	Rank
0	2.5	0	2.5
0	2.5	0	2.5
1	6.5	1	6.5
1	6.5	1	6.5
2	11	2	11
2	11	2	11
2	11	3	14.5
4	19	3	14.5
4	19	4	19
4	19	4	19
5	24.5	4	19
5	24.5	4	19
6	29	5	24.5
6	29	5	24.5
7	33.5	6	29

7	33.5	6	29
7	33.5	6	29
8	36.5	7	33.5
10	38	8	36.5
11	39	12	40
Sum	429	Sum	391

H_0: An American League team would score the same for games in which the designated hitter was used.
H_1: An American League team would score differently for games in which the designated hitter was used.

$U_1 = R_1 - \frac{n_1(n_1+1)}{2} = 429 - \frac{20*21}{2} = 219$
$U_2 = R_2 - \frac{n_2(n_2+1)}{2} = 391 - \frac{20*21}{2} = 181$

U = 181

Find the critical value using Figure 13-8, where $n_1 = 20$ and $n_2 = 20$. The critical value = 127.

Do not reject H_0, since U = 181 > CV = 127.

There is not enough evidence to support the claim that an American League team would score differently for games in which the designated hitter was used.

17. "Durable press" cotton fabrics are treated to improve their recovery from wrinkles after washing. "Wrinkle recovery angle" measures how well a fabric recovers from wrinkles. Higher is better. Here are data on the wrinkle recovery angle (in degrees) for a random sample of fabric specimens. A manufacturer wants to see if there is a difference in the wrinkle recovery angle for two different fabric treatments, Permafresh and Hylite. Test the claim using a 5% level of significance. Use the Mann-Whitney test.

H_0: There is no difference in the wrinkle recovery angle for two different fabric treatments, Permafresh and Hylite.
H_1: There is a difference in the wrinkle recovery angle for two different fabric treatments, Permafresh and Hylite.

Permafresh	Rank	Hylite	Rank
102	1	131	7.5
117	2	132	9
118	3	133	11
124	4	133	11
127	5	134	13
129	6	137	18

131	7.5	137	18
133	11	138	23
135	14	138	23
136	15	138	23
137	18	138	23
137	18	138	23
137	18	139	28
139	28	139	28
142	36	139	28
144	38	139	28
147	42	140	31.5
148	43.5	140	31.5
148	43.5	141	33.5
164	45	141	33.5
Sum	398.5	142	36
		142	36
		146	40
		146	40
		146	40
		Sum	636.5

$U_1 = R_1 - \frac{n_1(n_1+1)}{2} = 398.5 - \frac{20*21}{2} = 188.5$

$U_2 = R_2 - \frac{n_2(n_2+1)}{2} = 636.5 - \frac{25*26}{2} = 311.5$

U = 94.5

$$z = \frac{188.5-\left(\frac{20*25}{2}\right)}{\sqrt{\frac{20\cdot25(20+25+1)}{12}}} = -1.4048$$

Critical values = ±1.96

Do not reject H_0, since z = –1.4048 is not in the critical region.

There is not enough evidence to support the claim that there is a difference in the wrinkle recovery angle for two different fabric treatments, Permafresh and Hylite.

19. A movie theater company wants to see if there is a difference in the movie ticket sales in San Diego and Portland per week. They sample 20 sales from San Diego and 20 sales from Portland and count the number of tickets sold over a week. Use the Mann-Whitney test to test the claim using a 5% level of significance.

San Diego	Rank	Portland	Rank
182	1	209	3
206	2	211	4.5
211	4.5	212	6
214	8	214	8
215	10	214	8
217	12	216	11
219	15.5	218	13
221	20	219	15.5
221	20	219	15.5
223	24	219	15.5
226	28	220	18
229	31	221	20
231	32	222	22
232	33	223	24
233	34.5	223	24
234	36	224	26
235	37	226	28
239	38.5	226	28
239	38.5	228	30
243	40	233	34.5
Sum	465.5	Sum	354.5

H_0: There is no difference in the movie ticket sales in San Diego and Portland per week.
H_1: There is a difference in the movie ticket sales in San Diego and Portland per week.

$U_1 = R_1 - \frac{n_1(n_1+1)}{2} = 465.5 - \frac{20*21}{2} = 255.5$
$U_2 = R_2 - \frac{n_2(n_2+1)}{2} = 354.5 - \frac{20*21}{2} = 144.5$

$U = 144.5$

Do not reject H_0, since $U = 144.5 > CV = 127$.

There is not enough evidence to support the claim there is a difference in the movie ticket sales in San Diego and Portland per week.